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The Figure of Merit of Magnetic Amplifiers

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19 FEB 1953

REGULATING systems are a combination of devices whereby a large amount of energy is controlled by a small amount of energy. Large amounts of heat can be controlled by a small amount of thermal energy in a thermocouple. A feeble voltage can regulate the speed of a tandem mill; a tiny magnet governs the voltage of a large alternator; and a microwatt radar signal can guide a missile at supersonic speeds. It follows that power amplifiers play a major role in regulating systems; therefore, some of the principle characteristics of proportional power amplifiers will be examined.

Characteristics of Power Amplifiers

POWER AMPLIFICATION

Of primary importance in power amplifiers is the ratio of power output to power input of the amplifier, namely the power amplification. It is fundamental that the system power amplification must be such that the output power can be controlled by the available controlling energy; hence, the system power amplification is determined by the power output required, the power level of the detecting device, the nature of the disturbances to which the system is subjected, and the degree of regulation required.

RESPONSE

The rapidity with which an amplifier responds to an input signal is also of importance in determining the performance of regulating systems. Associated with the output power of a power amplifier is the storage of a certain amount of energy in the control elements of the amplifier. The response of the thermocouple is determined by the amount of heat stored in bringing it up to the proper temperature. Amplifiers utilizing magnetic fields store energy in the form of flux. A Silverstat contains energy stored in a stressed spring and moving inertia; and vacuum tubes are plagued with electrostatic energy stored in the interelectrode capacities. The phenomenon of changing the amount of stored energy for a change of input power prevents an instantaneous change in power output to conform to a change in input. This delay in response is frequently expressed as the time constant of the amplifier.

GAIN

A third important characteristic of power amplifier is gain, or, in the case of electric power amplifiers, voltage amplification. System gain governs the regulator's stiffness and, in conjunction with the amplifier's delays, determines the system response and stability. Unlike vacuum tubes, whose voltage gains are fairly well fixed, magnetic amplifiers, both static and rotating, enjoy flexibility, the voltage gains obtainable being limited mainly by the maximum and minimum wire sizes that may be used in the control and output circuits.

In designing regulating systems it is necessary to be able to evaluate the power amplification, delay, and gain of the amplifiers involved. In static and rotating magnetic amplifiers, because the gain is flexible the power amplification and delays define the capability of the amplifier. For this purpose the "figure of merit" of the amplifier has been defined as the ratio of power amplification to time constant. In this paper, it is demonstrated that a figure of merit of static magnetic amplifiers so defined exists, that it corresponds to one-half the ratio of power output to stored magnetic energy, and that it is calculable from the configuration of the amplifier. A method of designing magnetic amplifiers with high figure of merit also is presented. Methods of calculating the figure of merit of d-c rotating power amplifiers is presented elsewhere.

The Equivalent Circuit and Transfer Function of Magnetic Amplifiers

The performance of magnetic amplifiers in regulating systems may be expressed in the form of a transfer function. This function may be obtained by linearizing the magnetic amplifier and making certain simplifying assumptions. The simplified device may be understood and analyzed more easily by examining its equivalent circuit.

Therefore, a transfer function and an equivalent circuit for a self-saturating magnetic amplifier will be derived. The operation of the self-saturating magnetic amplifier has been described in the literature.[1] Briefly a control winding current establishes flux conditions in the magnetic structure, the flux conditions at the start of any positive half-cycle or supply voltage determining the load voltage. The control ampere-turns may take a series of valves, as shown in Figure 1, over a useful range *OB*, which determines the range of control ampere-turns necessary for utilization of the amplifier over its full range of output. This range of ampere-turns can be related to core properties, as illustrated in Figure 1, by the superimposed amplifier control characteristic and the operating curve of flux density versus field intensity.[2] Effects of rectifier leakage are neglected.

The relationship between control ampere-turns and the magnetic material and structure is

$$H_i=\frac{0.4\pi N_i I_i}{l}\ \text{(oersteds)} \qquad (1)$$

when l is the effective mean length of the core magnetic path in centimeters. The change in control H corresponding to a change in control ampere-turns OB is

$$\Delta H_i=\frac{0.4\pi N_i \Delta I_i}{l}\ \text{(oersteds)} \qquad (2)$$

ΔH_i is dependent on core material, lamination thickness, and supply fre-

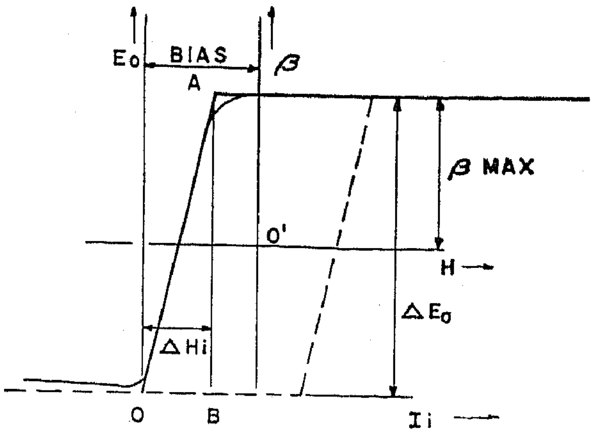

Figure 1. Operating β-H curve with superimposed control characteristic

—— Idealized control characteristic and left flank and top of operating β-H curve
—— Actual control characteristic
– – – Right flank and bottom of operating β-H curve

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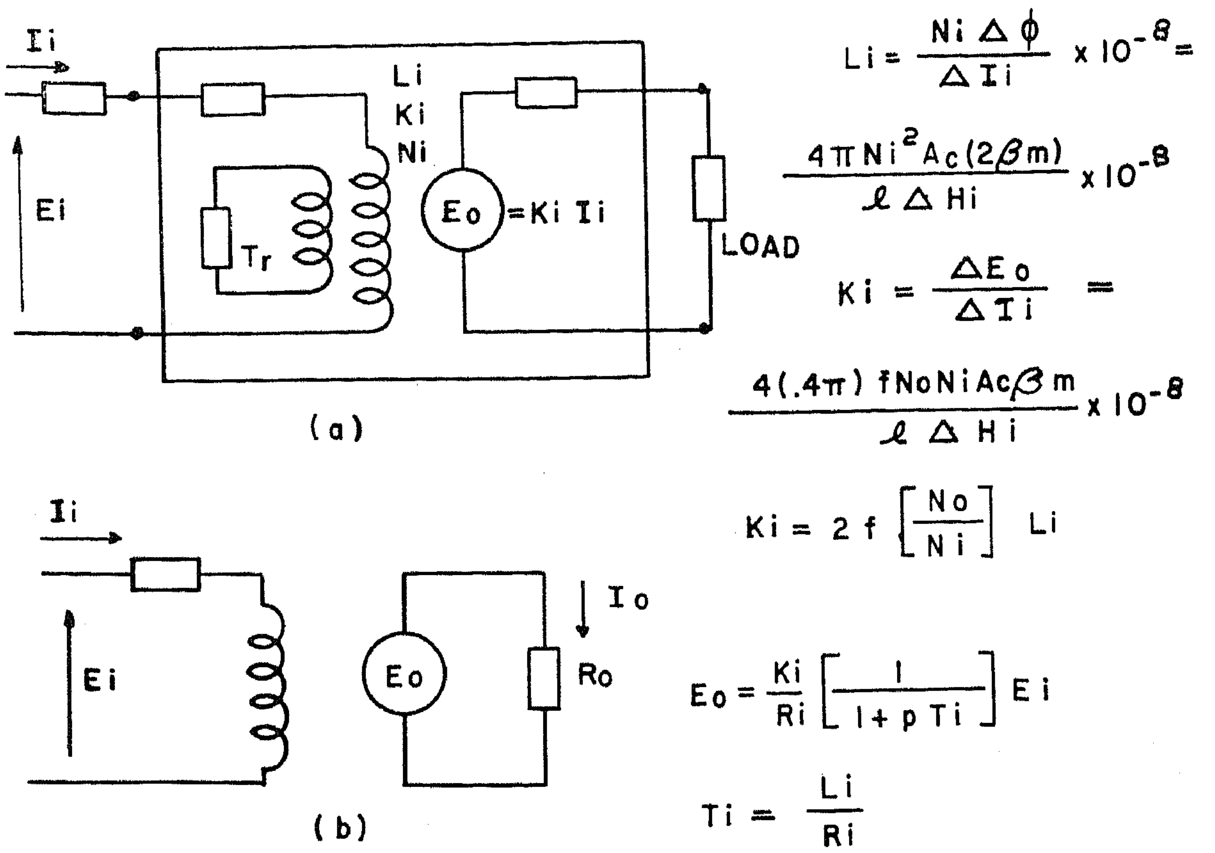


$$Li = \frac{Ni\,\Delta\phi}{\Delta Ii} \times 10^{-8} = \frac{4\pi Ni^2 A_c(2\beta m)}{\ell\,\Delta Hi} \times 10^{-8}$$

$$Ki = \frac{\Delta Eo}{\Delta Ii} = \frac{4(.4\pi)\,fNoNiAc\beta m}{\ell\,\Delta Hi} \times 10^{-8}$$

$$Ki = 2f\left[\frac{No}{Ni}\right] Li$$

$$Eo = \frac{Ki}{Ri}\left[\frac{1}{1+pTi}\right] Ei$$

$$Ti = \frac{Li}{Ri}$$

Figure 2(A). Equivalent circuit of self-saturating magnetic amplifier linearized over the curve OA. (B). Simplified equivalent circuit

quency. It is the change in control oersteds required to change the flux at the beginning of a positive half-cycle of the supply voltage over the left flank of the operating hysteresis loop.

The relationship between ΔE_o and β_m is

$$\Delta E_o = 4N_o f A_c \beta_m \times 10^{-8} \qquad (3)$$

where ΔE_o is the average value of the a-c supply voltage and the maximum output voltage of a single-phase amplifier.

Figure 2(A) is the equivalent circuit of the self-saturating magnetic amplifier shown in Figure 6. Figure 2(B) is a simplified equivalent circuit derived by lumping the resistances of the control circuit and the output circuit respectively and by neglecting the inductively coupled delay produced by the eddy currents and bias windings T_r. The transfer function is derived in Appendix I and is found to be as follows

$$E_o = \frac{K_i}{R_i} \frac{1}{1+p\frac{L_i}{R_i}} E_i \qquad (4)$$

where

E_o = the internal output voltage
E_i = input voltage
$K_i = \frac{\partial E_i}{\partial I_i}$
L_i = control current
R_i = control circuit resistance
L_i = control circuit inductance

Figure 3 is the equivalent circuit of a simple series-reactor-type magnetic amplifier over linear range of operation. The transfer function is derived in Appendix II and is found to be as follows

$$E_o = \frac{1}{1+p\frac{L_i}{R_i}}\left(\frac{K_i}{R_i}E_i - \frac{N_o}{N_i}K_i I_o\right) \qquad (5)$$

If the amplifier is loaded into a resistive load R_o and $(N_o/N_i)K_i \gg R_o$ which is generally the case when high performance magnetic-material is used, the following transfer function applies

$$E_o = \frac{N_i}{N_o}\frac{R_o}{R_i}\frac{1}{1+p\frac{N_i}{N_o}\frac{R_o L_i}{R_i K_i}} E_i \qquad (6)$$

When the amplifier is loaded into an inductive load (see Figure 4) the advantage of the negative current feedback is utilized, for the transfer function then becomes

$$E_o = \frac{R_o}{R_i}\frac{N_i}{N_o} \times \frac{1+p\frac{L}{R_o}}{1+p\,N_i R_o\left(\frac{L_i}{R_i}+\frac{L}{R_o}\right)+p^2\frac{N_i R_o}{N_o K_i}\frac{L_i L}{R_i R_o}} E_i \qquad (7)$$

The equivalent circuit of the parallel connected magnetic amplifier is shown in Figure 5 in which the delay produced by the a-c windings is included.

Energy Storage and the Figure of Merit

The figure of merit of a magnetic amplifier is defined as the ratio of power amplification to time constant

$$m = A_p/T \text{ per second} \qquad (8)$$

or alternately

$$m = A_p/T \text{ per cycle} \qquad (9)$$

where A_p is the amplifier power amplification and T is the amplifier time constant expressed in seconds, equation 8, or in cycles of the supply frequency, equation 9. Power amplification can be expressed

$$A_p = \frac{\Delta P_o}{\Delta P_i} = \frac{\Delta I_o^2 R_o}{\Delta I_i^2 R_i} \qquad (10)$$

the ratio of a change in output power to a change in input or control power. (For notation see Appendix III, Figure 6, and Figure 7.) The time constant of a magnetic amplifier is determined by the effective inductance of its control winding and the resistance of its control circuit which gives

$$T = L_i/R_i \text{ (seconds)} \qquad (11)$$

From equations 8, 10, and 11 is seen that

$$m = \frac{\Delta P_o}{L_i \Delta I_i^2} \text{ per second} \qquad (12)$$

or that the figure of merit of a magnetic amplifier is determined by the ratio of the change in amplifier power output to twice the change in energy stored in the amplifier control circuit. Equation 12 presents a clear physical picture of the operation of the power amplifier—a change in

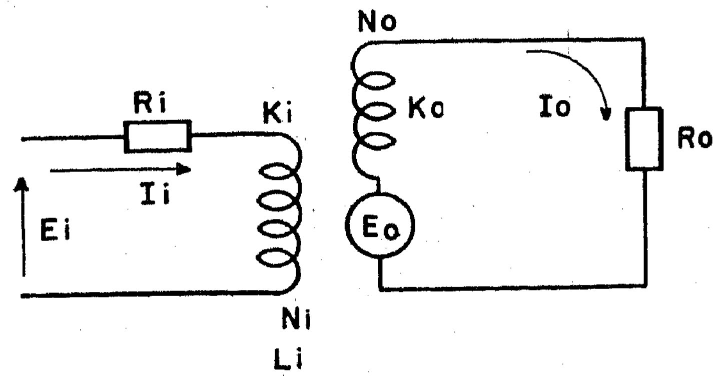
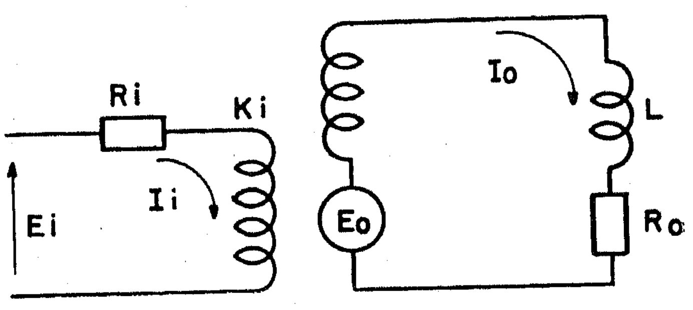


Figure 3 (left). Equivalent circuit of series-connected reactor magnetic amplifier with resistive load

Figure 4 (right). Equivalen circuit of series-connected reactor magnetic amplifier with inductive load



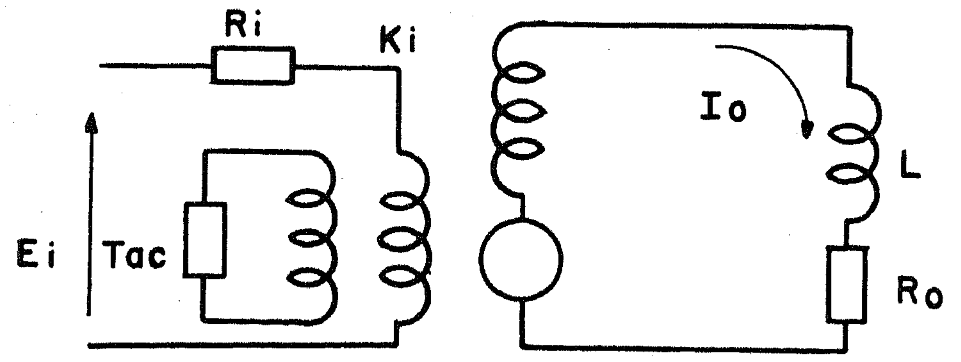


Figure 5 (left). Equivalent circuit of parallel-connected reactor magnetic amplifier with inductive load

control energy storage must accompany any change in output power.

Self-Saturating Magnetic Amplifier

Self-saturation in magnetic amplifiers refers to the circuitry of a group of basic single-phase and multiphase amplifiers. Amplifiers of this type have been described at some length in the literature. It has been customary in the literature to treat self-saturating amplifier circuits as simple reactor circuits with the addition of almost 100-per-cent positive feedback. When this positive feedback is added to a simple reactor circuit, the cyclic demagnetization of the reactors by the supply voltage no longer occurs and the control characteristic of the amplifier is primarily dependent on the characteristics of the reactor core material. The significant advance of the self-saturating circuit over the simple reactor circuit is in the ratio of power gain to amplifier time constant. A simple reactor circuit with a value of 3 for this ratio may have a ratio of 500 with the addition of self-saturating rectifiers. Since the self-saturating circuits do depend primarily on the characteristics of the reactor core material, they may be considered the basic magnetic amplifier circuits. In the same sense the simple reactor circuits can be considered as self-saturating circuits employing almost 100-per-cent negative current feedback.

The single-phase self-saturating circuits, see Figure 6, (the doubler, bridge circuit, and center-tapped transformer circuit) can be grouped and said to have very similar steady-state characteristics.[3] Their output power capabilities and required control ampere-turns are the same. The transient characteristics of this group differ in residual time constant which is dependent on specific circuitry. This is a time constant not associated with the amplifier control circuit and as such is not amenable to control circuit "forcing."

The 3-phase self-saturating circuits, see Figure 7, are combinations of single-phase circuits. The steady-state and transient characteristics of these circuits are dependent on the characteristics of their single-phase components and, as such, generalizations which apply to the single-phase component circuits will apply to the 3-phase circuits.

The figure of merit of a magnetic amplifier circuit

$$m=A_p/T \text{ per cycle} \tag{9}$$

is a constant for a given amplifier operating into a fixed load when residual time constant can be neglected. This expresses the experimentally proved fact that amplifier power gain and time constant are inversely related. One error in this relationship lies in the amplifier residual time constant. The residual time constant of the single-phase bridge and center-tapped transformer circuits is of the order of 0.5 cycle[4] of the supply frequency and of the order of 1 cycle for the doubler circuit. A second source of error lies in the phenomenon of commutation[5] at high values of control circuit resistance per control winding turns squared. When the amplifier operating time constant is high with respect to residual time constant, equation 9 defines a figure of merit which is substantially constant over a wide range of time constants and power amplifications.

To make the figure of merit concept useful, amplifier design conditions will be derived from it.

Single-Phase Magnetic Amplifier

PROPORTIONING OF REACTOR WINDOW AREA

The figure of merit of a magnetic amplifier bears a direct relationship to the relative proportions of reactor window area which can be devoted to the load winding and the window area which must be devoted to the control winding for a given measure of control. This relationship can be derived from the definition of m.

The control circuit time constant of a single-phase self-saturating magnetic amplifier is one-half the average voltage gain on a common turns base.[6]

$$T=1/2A_v \text{ (cycles)} \tag{13}$$

From equations 9 and 13

$$m=2A_I \text{ per cycle} \tag{14}$$

where A_I is amplifier average current gain on a common turns base and is measurable directly from the amplifier control characteristic.

From equation 14

$$\frac{\Delta N_o I_o}{\Delta N_i I_i}=\frac{m}{2} \tag{15}$$

The ratio of average output current to rms single reactor current is $1/\sqrt{2\mu}$ at maximum amplifier output. μ is dependent on load inductance and varies from 0.95 to 1 as the load varies from resistive to highly reactive. Assuming that output and control windings are worked at equal current densities, $m/2\sqrt{2\mu}$ is the ratio of load copper per core to control copper per core and is thus roughly equivalent to the ratio of window areas required by load to control winding. To obtain orders of magnitude, measured values of m are of the order of 200 to 1,500 in amplifiers employing square loop core materials, depending on actual core dimensions. For amplifiers of this type the reactor window area required by the control winding for full control is almost negligible as compared to that required by the load winding.

RELATION OF CORE DIMENSIONS AND CHARACTERISTICS TO FIGURE OF MERIT

Amplifier core dimensions and core material characteristics bear a very important relationship to the figure of merit. It is desirable in many cases to design the amplifier core and windings, the only independent valuables that the amplifier designer has readily at hand, to conform

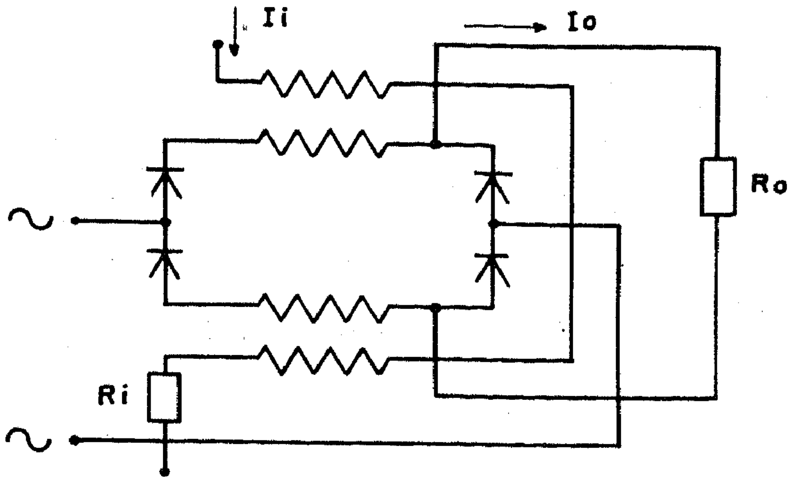


Figure 6 (right). Single-phase self-saturating magnetic amplifier—bridge circuit

R_o = lumped load circuit resistance

R_i = lumped control circuit resistance

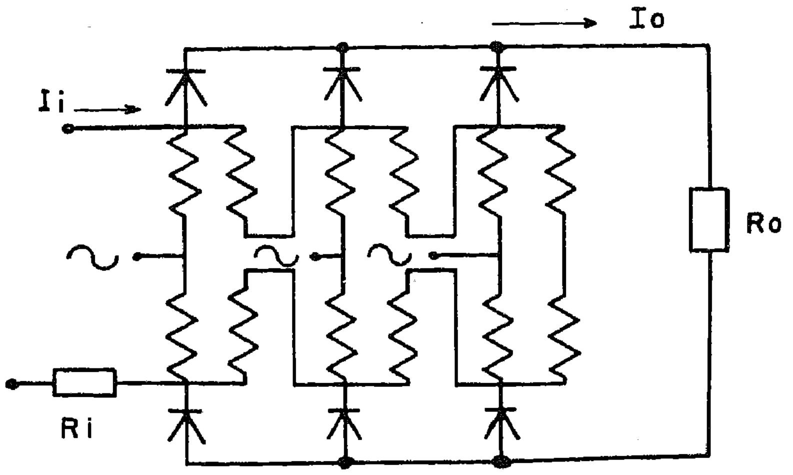


Figure 7. Three-phase self-saturating magnetic amplifier—full-wave bridge circuit

with this relationship—in other words, to maximize the amplifier figure of merit.

Corresponding to ΔH_i change in control oersteds, load current changes from approximately zero, magnetizing current only, to the value corresponding to full amplifier saturation.

$$\Delta I_o = \frac{\Delta E_o}{R_o} - I_o \text{ mag} \tag{16}$$

or

$$\Delta I_o \approx I_o \text{ max} \tag{16A}$$

where ΔE_o is the average value of the supply voltage and R_o is the total output circuit resistance.

From equations 2 and 14

$$m = 2I_o \text{ max } N_o \frac{0.4\pi}{\Delta H_i l} \text{ per cycle} \tag{17}$$

I_oN_o can be expressed in terms of core dimensions from the considerations outlined in the previous section on "Proportioning of Reactor Window Area." In Appendix IV an expression is derived from m in terms of readily determinable core constants and dimensions. This expression is

$$m = \frac{2\sqrt{2\mu} s A_w}{\rho} \frac{0.4\pi}{\Delta H_i l} \text{ per cycle} \tag{18}$$

As defined in Appendix I,

s is the load winding space factor
A_w is the window area of the core in circular mils
ρ is the circular mils per rms amperes for the load winding at full output

When applied to a particular amplifier circuit employing a particular core structure, equation 18 shows the required design considerations. Equation 18 is applied to two important amplifier core structures, the rectangular and toroidal structures in Appendix V. Some immediate generalizations are possible:

1. m is independent of the saturation flux density of the core material.
2. m is independent of stack height for stacked cores or strip width for toroidal cores.
3. m is maximized when core build up is minimized.
4. m increases as a linear dimension of the core window—in toroidal cores as the inside diameter is increased, in rectangular cores as window width and length is increased.
5. Since a circle is capable of a greater ratio of area to circumference than a rectangle, toroidal construction will give the maximum value of m, other factors being equal. The comparative ratio is $4/\pi = 1.3/1$, comparing a toroidal with a square core with equal control requirements for the theoretical case of zero core build-up.
6. m is maximized for maximum load winding space factor and maximum working current density of load copper.
7. m is inversely proportional to ΔH_i.

These generalizations of course are dependent on the assumptions of the derivation of equation 18. These assumptions are:

1. Minimum amplifier output is essentially zero when compared with maximum output.
2. Amplifier control characteristic is linear from minimum output to maximum output.
3. ρ and s do not change appreciably for changes in core geometry for a given amplifier output rating. Practically extremes in geometry are limited by the variations in ρ and s.

Equations 14 and 18 are checked against the measured value of m for a toroidal core self-saturating magnetic amplifier in Appendix VI.

Three-Phase Full-Wave Magnetic Amplifier

PROPORTIONING OF REACTOR WINDOW AREA

The control circuit response time of a 3-phase full-wave magnetic amplifier is

$$T = A_v/3 \text{ (cycles)} \tag{19}$$

This expression can be used with equation 2 to give

$$m = 3A_I \text{ per cycle} \tag{20}$$

where A_I is again amplifier average current gain on a common turns base, or

$$\frac{\Delta N_o I_o}{\Delta N_i I_i} = \frac{m}{3} \text{ per cycle} \tag{21}$$

The ratio of average output current to rms single reactor current for the 3-phase bridge circuit is $1/\sqrt{3}$. Assuming again that output and control windings are worked at equal current densities, $m/3\sqrt{3}$ is the required ratio of load copper per core to control copper per core.

Similarly, ratios of output winding copper area to control winding copper area can be calculated for other 3-phase circuits.

RELATION OF CORE DIMENSIONS AND CHARACTERISTICS TO FIGURE OF MERIT

An expression for m similar to that of equation 18 can be derived for the full-wave 3-phase amplifier circuit. This expression is

$$m = \frac{3\sqrt{3} s A_w}{\rho} \frac{0.4\pi}{\Delta H_i l} \text{ per cycle} \tag{22}$$

where the terms are as defined before. Similarly an expression for m can be derived for other 3-phase circuits. The gen-

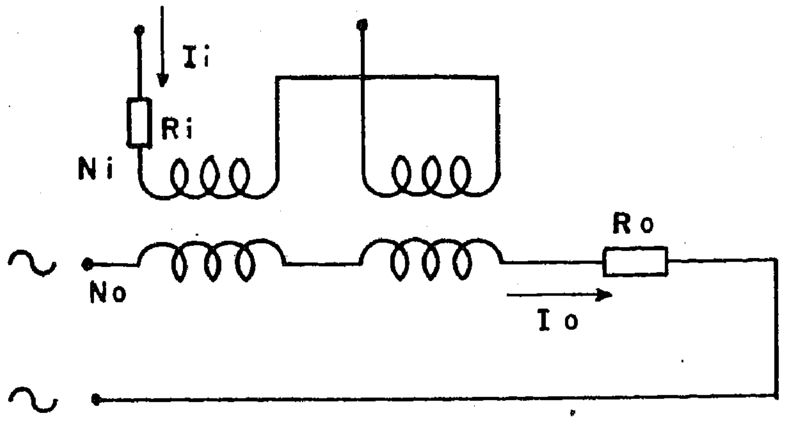
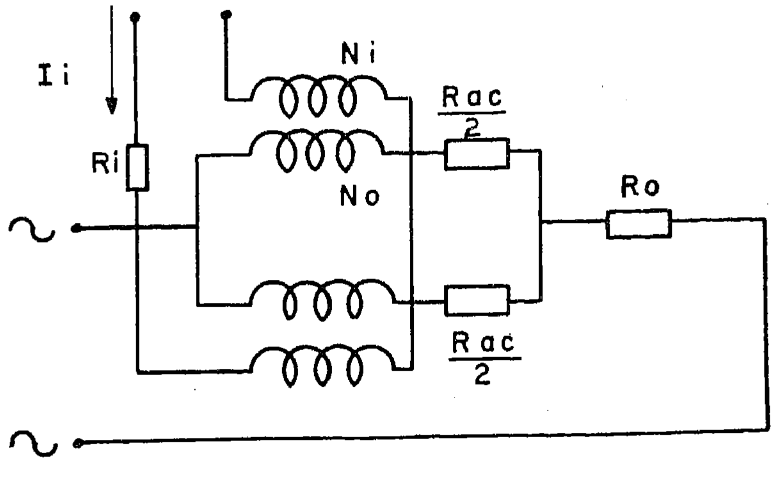


Figure 8 (left). Series-connected reactor magnetic amplifier

Figure 9 (right). Parallel - connected reactor magnetic amplifier



eralizations drawn previously are again true within the limits of the assumptions of the derivation.

Simple Reactor Magnetic Amplifier

SERIES-CONNECTED REACTORS

The time constant of the magnetic amplifier consisting of two series-connected reactors controlling a resistive load, as shown in Figure 8, has been derived by Storm.[7] The time constant in cycles is given by

$$T=\frac{1R_o}{4R_i}\,\frac{N_i^2}{N_o^2}\text{ per cycle} \qquad (23)$$

neglecting residual delay associated with the load circuit and eddy currents. Power gain of the amplifier can be derived from the equal ampere-turns relationship which governs simple reactor control characteristics over the control range. This relationship can be expressed as

$$N_oI_o=N_iI_i\text{ (ampere-turns)} \qquad (24)$$

$$\frac{(I_o)^2}{(I_i)^2}=\frac{(N_i)^2}{(N_o)} \qquad (25)$$

and

$$A_p=\frac{(I_o)^2R_o}{(I_i)^2R_i}=\left(\frac{N_i}{N_o}\right)^2\frac{R_o}{R_i} \qquad (26)$$

From equation 23 the figure of merit can be obtained

$$m=\frac{A_p}{T}=\frac{\frac{R_o}{R_i}\left(\frac{N_i}{N_o}\right)^2}{1/4\,\frac{R_o}{R_i}\left(\frac{N_i}{N_o}\right)^2}=4\text{ per cycle} \qquad (27)$$

Thus the figure of merit m for a simple series reactor magnetic amplifier, when operating into a resistive load, is a constant numerically equal to 4 per cycle. When operating into an inductive load the negative current feedback of the reactor should be considered as outlined in the consideration of equivalent circuits.

PARALLEL-CONNECTED REACTORS

The time constant of the magnetic amplifier consisting of two parallel-connected reactors controlling a resistive load, as shown in Figure 9, is not easily expressible in terms of load and control circuit resistances and turns as in the case of the series-connected reactors. The time constant in cycles can be expressed[8] as

$$T=\frac{R_o}{R_i}\left(\frac{N_i}{N_o}\right)^2(1+\alpha)\text{ per cycle} \qquad (28)$$

where α is the ratio R_{ac}/R_o, again neglecting residual delay.

R_{ac} is defined as the sum of the resistances of reactor load windings. Power gain again can be derived from the ampere-turns relationship over the useful control region.

$$I_iN_i=1/2I_oN_o\text{ (ampere-turns)} \qquad (29)$$

As for the series-connected reactors, the figure of merit can be calculated as

$$m=A_p/T=4/(1+\alpha)\text{ per cycle} \qquad (30)$$

It is seen that the figure of merit for the parallel connected reactor amplifier is always less than for the series-connected reactor amplifier by the ratio $1/(1+\alpha)$.

Conclusions

FIGURE OF MERIT SELF-SATURATING MAGNETIC AMPLIFIER

1. The figure of merit and equivalent circuits are determinable from core geometry and characteristics.
2. The figure of merit is determinable experimentally by static test.
3. Correlation of figure of merit as determined by the methods described in "Proportioning of Reactor Window Area" and "Relation of Core Dimensions and Characteristics to Figure of Merit" check very closely.

FIGURE OF MERIT SIMPLE REACTOR MAGNETIC AMPLIFIER

1. The figure of merit for resistive load is independent of core geometry and within wide limits independent of core characteristics.
2. Core geometry and characteristics determine current forcing effect when operating into inductive loads.
3. The simple reactor amplifier is useful when output current independent of output voltage is desired.

Appendix I. Transfer Function of Self-Saturating Magnetic Amplifier

Refer to Figure 2

$$E_i=I_iR_i+p\frac{L_i}{K_i}E_o$$

$$E_o=K_iI_i$$

$$E_o=\frac{K_i}{R_i}\,\frac{1}{1+p\frac{L_i}{R_i}}E_i$$

where

$$K_i=\frac{\partial E_o}{\partial I_o}=\frac{\Delta E_o}{\Delta I_o}=\frac{4N_ofA_c\beta_m\times10^{-8}}{l\Delta H_i/0.4\pi N_i}$$

$$=\frac{1.6\pi fN_oN_iA_c\beta_m\times10^{-8}}{l\Delta H_i}$$

$$L_i=\frac{N_i\phi}{I_i}\times10^{-8}=\frac{N_iA_c(2\beta_m)\times10^{-8}}{l\Delta H_i/0.4\pi N_i}$$

$$=\frac{0.8\pi N_i^2A_c\beta_m\times10^{-8}}{l\Delta H_i}$$

$$K_i=2f(N_o/N_i)$$

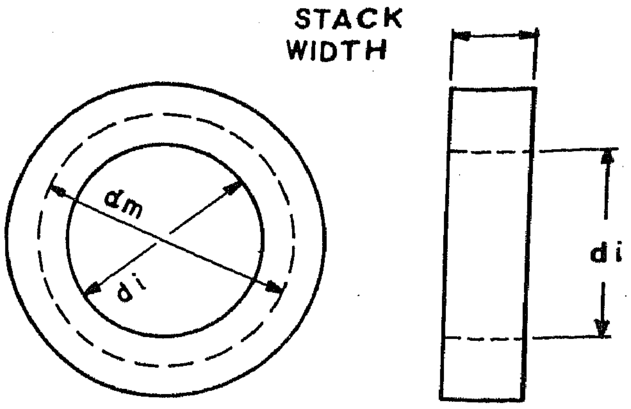


Figure 10. Toroidal core

Appendix II. Transfer Function of Simple Reactor Magnetic Amplifiers

Refer to Figure 7

$$K_i=\frac{\partial E_o}{\partial I_i}\qquad K_o=\frac{\partial E_o}{\partial I_o}$$

If $R_o=0$, then $E_o=0$ and $I_o=(N_o/N_i)I_i$.
Therefore $K_o=(N_o/N_i)K_i$.
From reference 7

$$I_o=I_i+\frac{E}{\omega L_{ac}}$$

where E=average voltage across reactor

$$\frac{\partial I_o}{\partial E_o}=\frac{1}{K_o}=\frac{1}{\omega L_{ac}}\quad\frac{\partial E}{\partial E_o}=\frac{1}{\omega L_{ac}}$$

L_{ac}=a-c inductance of one unsaturated reactor

$$K_o=\omega L_{ac}$$

$$K_o=\frac{2\pi fN_o0.2A_c\beta_m\times10^{-8}}{l\Delta H_i/0.4\pi N_o}$$

$$K_o=\frac{1.6fN_o^2A_c\beta_m\times10^{-8}}{l\Delta H_i}$$

$$E_i=R_iL_i+p\frac{L_o}{K_i}E_o$$

$$E_o=K_i\left(I_i=\frac{N_o}{N_i}I_o\right)$$

$$E_o=\frac{1}{1+p\frac{L_i}{R_i}}\left(\frac{K_i}{R_i}E_o-\frac{N_o}{N_i}K_iI_o\right)$$

If the load is resistive, $I_o=E_o/R_o$

$$E_o=\frac{K_i}{R_i}\,\frac{1}{\left(1+\frac{N_oK_i}{N_iR_o}\right)}\,\frac{1}{1+p\frac{L_i}{R_i}}\,\frac{1}{1+\frac{N_oK_i}{N_iR_o}}E_i$$

if $\frac{N_oK_i}{N_iR_o}\gg1$

$$E_o=\frac{R_o}{R_i}\,\frac{N_i}{N_o}\,\frac{1}{1+P\frac{L_iN_iR_o}{R_iN_oK_i}}E_i$$

$$A_v=\frac{R_oN_i}{R_iN_o}\qquad T=\frac{L_iN_iR_o}{R_iN_oK_i}$$

$$T=\frac{1}{4f}\,\frac{R_o}{R_i}\left(\frac{N_i}{N_o}\right)^2$$

$$\frac{L_i}{K_i}=\frac{1}{4f}\,\frac{N_i}{N_o}$$

$$K_i=\frac{N_i}{N_o}K_o=\frac{1.6\pi f N_o N_i A_c \beta_m \times 10^{-8}}{l\Delta H_i}$$

$$L_i=\frac{0.4\pi N_1{}^2 A_c \beta_m \times 10^{-8}}{l\Delta H_i}$$

If the load is inductive, as shown in Figure 8

$$I_o=\frac{E_o}{R_o\left(1+p\,\dfrac{L}{R_o}\right)}$$

$$E_o=\frac{1}{1+p\,\dfrac{L_1}{R_1}}\left(\frac{K_1}{R_1}E_1-\frac{N_o}{N_1}\,\frac{K_1}{R_o}\,\frac{E_o}{\left(1+p\,\dfrac{L}{R_o}\right)}\right)$$

$$E_o=\frac{K_i}{R_i}\,\frac{1}{1+\dfrac{N_oK_i}{R_oN_i}}\times$$

$$\frac{1+pL/R_o \quad E_i}{1+p\dfrac{1}{1+\dfrac{N_oK_i}{N_iR_o}}\left(\dfrac{L_i}{R_i}+\dfrac{L}{R_o}\right)+ \qquad p^2\dfrac{1}{1+\dfrac{N_oK_i}{N_iR_o}}\,\dfrac{L_iL}{R_iR_o}}$$

$$\frac{N_oK_i}{R_oN_i}>>1$$

$$E_o=\frac{R_o}{R_o}\,\frac{N_i}{N_o}\times$$

$$\frac{(1+pL/R_o)E_i}{1+p\dfrac{N_iR_o}{N_oK_i}\left(\dfrac{L_i}{R_i}+\dfrac{L}{R_o}\right)+p^2\dfrac{N_iR_o}{N_oK_i}\,\dfrac{L_i}{R_i}\,\dfrac{L}{R_o}}$$

Appendix III. Definition of Terms

m = ratio of power amplification to time constant, per cycle or per second

A_p = ratio of change in amplifier output power to change in input power

T = amplifier 63 per cent response time, cycles or seconds

P_o = total amount of power dissipated in amplifier output circuit, watts

P_i = total amount of power dissipated in amplifier control circuit, watts

I_o = amplifier output or load current, average amperes

I_i = amplifier input or control current, average amperes

N_o = amplifier output winding turns per core

N_i = amplifier control winding turns per core

R_o = total resistance in amplifier output circuit, ohms

R_i = total resistance in amplifier input or control circuit, ohms

$\Delta P_o=(\Delta I_o)^2R_o$, watts

$\Delta P_i=(\Delta I_i)^2R_i$, watts

l = effective mean length of core magnetic path, centimeters

A_v = amplifier average voltage gain on a common turns base

A_I = amplifier average current gain on a common turns base

A_c = core cross-sectional area, square centimeters

A_w = core window area, circular mils

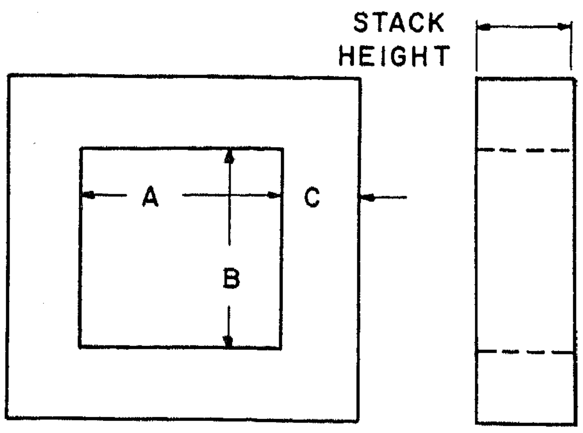


Figure 11. Square core

Appendix IV. Figure of Merit Related to Core Dimensions

Single-Phase Self-Saturating Magnetic Amplifier

$$m=2I_o\max N_o\frac{0.4\pi}{\Delta H_i l}\text{ per cycle} \tag{17}$$

It is now necessary to relate I_o max N_o to core dimensions. The maximum value of rms reactor current which essentially determines reactor heating at power frequencies is $I_o/\sqrt{2\mu}$. The maximum rms ampere-turns per reactor are then $I_oN_o/\sqrt{2\mu}$. Ampere-turns are directly related to copper and consequently to reactor window area.

For a given reactor volt-ampere rating the reactor load winding conductor can be assigned a minimum value of cross section per maximum rms ampere at a given supply frequency. This value ρ, the minimum permissible circular mils per rms ampere, is related to volt-ampere rating since reactors of a given volt-ampere rating at a given supply frequency have approximately the same heat dissipation capabilities.

$\rho I_oN_{o\,max}/\sqrt{2\mu}$ is equal to the reactor window area in circular mils times the load winding space factor.

$$\frac{\rho I_oN_{o\,max}}{\sqrt{2\mu}}=A_ws\text{ (circular mils)}$$

where s is the ratio between load winding copper area and total reactor window area.

Equation 16 now can be obtained from

$$I_oN_{o\,max}=\frac{\sqrt{2\mu}}{\rho}A_ws\text{ ampere turns}$$

$$m=2\sqrt{2\mu}\,\frac{A_ws}{\rho}\,\frac{0.4\pi}{\Delta H_il}\text{ per cycle} \tag{18}$$

Appendix V. Figure of Merit Related to Core Dimensions

Toroidal and Square Cores Single-Phase Self-Saturating Magnetic Amplifiers

$$m=2\sqrt{2\mu}\,\frac{A_ws}{\rho}\,\frac{0.4\pi}{\Delta H_il}\text{ per cycle} \tag{18}$$

Toroidal Core Construction (Figure 10)

$A_w=di^2\times10^6$ (centimeters)

where di is core inside diameter in inches.

$l=2.54\pi\,dm$ (centimeters)

where dm is core mean diameter in inches.

Then

$$m=2\sqrt{2\mu}\frac{sdi^2}{\rho}\times10^6\,\frac{0.157}{\Delta H_idm}\text{ per cycle} \tag{18A}$$

Rectangular Core Construction (Figure 11)

$A_w=4/\pi ab\times10^6$ (circular mils)

where a and b are in inches.

$l=(2a+2b+4c)2.54=2(a+b+2c)2.54$

where a, b, and c are in inches.

Then

$$m=\frac{8\sqrt{2\mu}}{\pi}\,\frac{s}{\rho}\,ab\times10^6\times\left[\frac{0.08\pi}{\Delta HI\,(a+b+2c)}\right]\text{ per cycle} \tag{18B}$$

For the special case of $a=b$ or a square window area

$$m=\frac{8\sqrt{2\mu}}{\pi}\,\frac{s}{\rho}\times10^6\left[\frac{0.04\pi}{\Delta H_i(a+c)}\right]\text{ per cycle} \tag{18C}$$

Appendix VI. Experimental Check of Equations 14 and 18

$$m=2A_I\text{ per cycle} \tag{14}$$

$$m=2\,\frac{\sqrt{2\mu}s}{\rho}A_w\,\frac{0.4\pi}{\Delta H_il}\text{ per cycle} \tag{18}$$

Amplifier to be checked has the following characteristics:

Core material, Hipernik V
Core dimensions (toroidal)
  Outside diameter, 2$^1/_2$ inches
  Inside diameter, 1$^1/_2$ inches
  Strip width, $^1/_2$ inch
N_o, 2,300 turns number 26 American Wire Gauge

Two cores are connected in a single-phase self-saturating circuit giving these measured characteristics:

Rated supply voltage, 115 volt, 60 cycles per second
Rated power at the load, 30 watts
Rated load resistance, 125 ohms
Amplifier internal resistance, 60 ohms
Measured current gain on common turns base, 575
ΔH_i, 0.18 oersted
di, 1.5 inches
dm, 2 inches
Rated load current I_o, 0.56 ampere

Operating at $N_i{}^2/R_i=875$

$T=10$ cycles measured

$A_p=10{,}200$

$T_r=2.1$ cycles measured

From equation 9

$$m=\frac{A_p}{T}$$

$$=\frac{10{,}200}{10\text{-}2.1}=1{,}290\text{ per cycle}$$

From equation 14

$$m=2A_I$$

$m=2(575)=1{,}150$ per cycle

To check equation 18, ρ and s must be calculated from the available data.

Number 26 American Wire Gauge has 253 circular mils

$$\rho=\frac{253}{0.56}\sqrt{2(0.95)}=575 \text{ circular mils per rms ampere}$$

$$s=\frac{253\times2300}{(1.5)^2\times10^6}=0.26$$

$$m=2\sqrt{2\mu s}\,\frac{di^2}{\rho}\times10^6\,\frac{0.157}{\Delta H_i dm} \text{ per cycle} \tag{18A}$$

$$m=2\sqrt{2(0.95)}(0.26)\,\frac{(1.5)^2}{575}\times10^6\left(\frac{0.157}{0.18\,(2)}\right)$$

$=1{,}145$ per cycle

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No Discussion

Automatic Switching for Nation-wide Telephone Service

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Synopsis: A plan for automatic long-distance switching, which ultimately will embrace the entire area of the United States and extend into Canada and perhaps Mexico has been formulated and important steps have been taken toward its realization. The plan contemplates that when a telephone customer places a call with a long-distance operator, this operator will be able to establish a connection to any desired telephone simply by playing a 10- or 11-digit code into an automatic mechanism. She will receive distinctive signals when the called telephone answers or when the telephone or the toll circuits are busy. Thus she will control completely the establishment of the connection and will have available to her the information necessary for proper billing of the call. The plan also contemplates that telephone customers ultimately will be able to dial long-distance calls themselves, wherever may be the locations of the calling and called telephones.

EVER SINCE the invention of the telephone 76 years ago, development work has been pressing forward both in telephone transmission and in switching. These two fields have been closely interrelated in the development of telephone service on a nation-wide basis, and neither could have progressed as it has without corresponding progress in the other.

The first development of equipment for the mechanical switching of telephone lines was the local dial system to enable one customer to be connected with another in the same town. It was a natural step to develop the equipment so that operators in nearby towns could complete toll calls through this local dial equipment. This was done first by using the local equipment and then with progressive modifications making it more and more suitable for toll.

By these means through the decades of the 1920's and 1930's regional networks were developed for operator toll dialing, using step-by-step types of equipment, particularly in Southern California, Connecticut, and Ohio. Also many short-haul toll calls in metropolitan areas were handled in connection with the panel-type dial equipment which was developed for automatic switching in these areas.

Also during this period the range of customer dialing in large metropolitan areas was extended, where local service is measured by message registers, through arrangements for the multiple registration of calls for which the charge was more than one local unit.

An important feature of switching development in this period was the perfecting of common control switching systems for large metropolitan areas endowed with a high degree of intelligence and great reliability.[1] As will be shown, still more extensive and complicated functions must be performed by the common control systems of a nationwide automatic switching system.

Also throughout this period great advance was made in the quality and stability of long-distance circuits. Telephone connections, some with as many as five circuits in tandem, were being established regularly by telephone operators with satisfactory over-all transmission. The limitation was in the speed and accuracy with which multiple switches could be made by operators rather than in the over-all transmission characteristics.

Several factors have worked together to bring about a big expansion of long-distance telephone service. These include the great growth in the number of telephones in service, improvements in long-distance transmission, in switching, and in methods of traffic operation. Since automatic switching becomes increasingly attractive as the traffic density increases, this large growth pointed toward the desirability of further mechanizing the switching operations.

In 1943 there was cut into service in Philadelphia the first installation of the number 4 toll crossbar system.[2] This system was designed to enable general automatic switching of toll connections in and out of large metropolitan areas and had many of the capabilities necessary for nationwide switching.

The various considerations already mentioned, coupled with the success of the number 4 installation at Philadelphia, led to studies of the service and operating results which might be expected from a nation-wide extension of automatic switching. The conclusion was reached that this would be a desir-

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able objective of the Bell System companies and would result in a very substantial further improvement in the speed and accuracy of handling of long-distance messages. Accordingly, during the next few years, a national plan was prepared and was adopted by the telephone companies.

General Plan for Nation-wide Automatic Switching

The features of this nation-wide plan and the present status of its application form the subject of three companion papers.[3–5] The basic requirements to be met in the development of this plan included the following:

1. It should be suitable for the nation-wide extension of automatic switching both by originating toll operators and by the customers direct.

   When this work was commenced it was clear that a program leading toward general nation-wide operator dialing was desirable. Subsequent developments have confirmed the wisdom of making the basic plan consistent with general nation-wide customer dialing as well, since it now appears that a very wide extension of this form of service will become desirable.

2. The plan must provide for satisfactory over-all service between any two telephones in this country and Canada.

   Under manual operation satisfactory over-all service was provided for by the general toll switching plan in use since about 1930. This plan is modified to recognize the far greater speed and accuracy of automatic switching compared with manual switching. This involves also modifications of transmission design standards so that the over-all connections will continue to be satisfactory.

3. The system must be designed for instantaneous service, so that delays due to lack of circuits or equipment would be very infrequent. This is necessary, both from the standpoints of service and the avoidance of tieups, particularly of the automatic switching machinery.

   A trunking system must, therefore, be devised which will meet most economically this requirement, considering over-all costs of lines, switching equipment, and operation.

4. Machines must be designed for use at strategic points in the network, called control switching points, to perform automatically the various tasks required to make the over-all plan operative and economical.

5. The entire plan must be such as to provide satisfactorily for growth, for flexibility to meet changing conditions, and for minimum over-all costs of operation.

Fundamental Plans for Toll Plant

Mr. Pilliod's paper[3] discusses the fundamental layout of plant for nation-wide operator toll dialing. This is subject to changes from time to time with further specific studies, as is the case with all far-reaching fundamental plans of this type. The additional requirements imposed by nation-wide customer dialing are still under study as will be discussed a little later.

The national toll switching plan is modified so that there may be a maximum of eight toll circuits switched together to connect any two telephones compared with the previous limit of five.[6] In order to handle the entire traffic of the country, approximately 100 control switching points are necessary at which highly intelligent common control switching systems of the number 4 crossbar-type will be placed.

A very important feature of the layout is a trunking plan providing for a high degree of use of alternate routes. To design all of the toll circuit groups of the country for a no-delay service would be very expensive. However, taking advantage of the extreme rapidity of automatic switching and the ability to build into the machine capacity for using a large number of alternate routes, a trunking system has been devised in which only about one-sixth the toll circuit groups of the country need be engineered on a very liberal basis. These are called final groups and are the groups to which the machine ultimately appeals if all of the more direct circuit groups are busy. These more direct circuit groups can then be engineered on a basis providing for high usage of the circuits, recognizing that when one group is busy the machine appeals to another and so on until as a last resort the final group is used.

In determining means for handling all of the toll messages with a relatively small number of control switching points, tremendous advantage was derived from modern transmission developments, particularly carrier systems which give a great economy from the concentration on a long-distance route of large numbers of telephone circuits—numbers often running into the thousands. As a result, a considerable degree of circuitous routing and back hauling of circuits is economical if by these means the circuits can be concentrated on heavy routes. This in turn lends itself to a plan using a minimum of control switching points.

Nation-wide Numbering Plan

In the previous use of automatic switching by toll operators, the operators were furnished with codes by means of which could be selected the various circuits necessary to reach the destination. These codes were dialed, followed by the local number of the called party. With this system, toll operators calling a given telephone from different remote cities would, in general, use different codes corresponding to the different circuit groups which they must select.

For nation-wide toll dialing even by operators this system would have impossible complications, and for nation-wide customer dialing it is clear that the code to be dialed must represent uniquely the office which serves the called telephone and that office only and not be dependent upon the route to be followed to reach it. In other words, it involves the development of what is called a destination-type code. Another description of this code plan is to say that for toll dialing purposes each telephone in the country (and Canada) must have a distinctive telephone number different from that of every other telephone.

It is also clear that as a practical matter this number should be based upon the local telephone number of the customer prefixed by a minimum number of digits, following easily understood rules.

To bring this about has involved a very high order of planning. Such a plan has been perfected and currently forms the basis for the determination of the coding of all new telephone offices and for changes in office codes when these are necessary. The development of this is the subject of Mr. Nunn's paper.[4]

Customer Toll Dialing

When the customer is to dial long-distance calls directly without assistance from any operator, two additional requirements are imposed beyond those necessary for nation-wide operator dialing.

1. The customer normally is connected to a local central office, but for the purpose of nation-wide toll dialing he must be connected to the nation-wide toll network. At present he does this by dialing a code such as "211" which connects him with the long-distance operator. This procedure could be continued. However, since the customer must in any event dial 10 digits for the longest hauls to designate the called telephone, it is desirable if possible to cut out this preliminary step. That would mean modifying the local central office equipment so that it would receive the 10 digit numbers and transmit them on to the toll equipment. This is a simple undertaking for local central offices using the latest type of local central office equipment, called number 5 crossbar, which was designed with this in view.[7] For older types of equipment, the job is more difficult.

2. The switching equipment must be provided with automatic means for recording

all of the information necessary for charging the call. In the case of operator dialing this is now done manually by the operator.

Great advances have been made in recent years in the development of automatic message-recording equipment. In 1944 there was placed in service in California the first installation in this country of automatic ticketing equipment.[8] This equipment is associated with step-by-step local switching equipment and automatically prints for each call a ticket similar to that prepared by the operator with manual operation. In 1948 there was installed in Media, near Philadelphia, a greatly improved type of message recording equipment in which the information appears in the form of punched holes in a tape.[9] This equipment is much more economical than the earlier system and also lends itself to the automatic preparation of toll statements or bills.

The present forms of equipment have been designed to be associated with local central offices. A careful study has been made of their field of application and the basic plan necessary to provide for a general nation-wide extension of customer dialing. This indicates that there will be a large field for automatic message accounting equipment associated with the toll network and arranged to receive orders for toll messages from a number of local dial offices. This centralized Automatic Message Accounting equipment, as it is called, is under development and an initial installation will be made next year in Washington, D. C. In this installation the range of customer dialing will be limited and certain service features will be lacking, which it is planned to add later.

The nation-wide extension of customer toll dialing involves many operating problems in addition to those relating to the design of the plant. These problems involve the extent to which customers wish to dial long-distance calls, requiring 10 pulls of the dial, the accuracy of dialing, the treatment of wrong numbers, provision for giving subscribers information regarding telephone numbers in distant cities, information on charges, and many other questions.

Recognizing that the best way to develop these questions is a trial, arrangements were made to open such a trial last fall at Englewood, N. J. This office is equipped with a number 5 crossbar system so that arrangements for such a trial could be made readily there. The Englewood customers are able to dial directly any of about 11,000,000 telephones in 10 metropolitan areas scattered throughout the country, including Boston, New York, Pittsburgh, Cleveland, Chicago, and San Francisco and the Bay area.

The results of this trial have been very encouraging. Subscribers are continuing to dial over 95 per cent of all the calls which can be dialed. Errors due to wrong numbers are at a minimum and other difficulties are relatively low. In so far as this trial can answer the questions, the results are all in favor of the nation-wide extension of customer dialing as the development and installation of facilities suitable for this purpose make it possible to do so.

In view of the prospect of nation-wide customer dialing, fundamental plan studies are now being made by the Telephone Companies throughout the country of the whole layout of plant including the distribution of centralized automatic message accounting equipments with the future general application of this method of operation. The present indication is that the number of points at which toll-operating centers will be required will be greatly reduced. This will react in important ways on the design of telephone buildings, telephone equipment installations, and toll circuit routes.

Automatic Toll Switching and Accounting Equipment

All of these plans depend upon the successful development of striking innovations in toll switching and automatic message accounting equipments. The plans in turn react upon the features to be incorporated in such equipments and upon the schedule of their development. Mr. Shipley's paper[5] tells about the more important features of these equipments and the problems which are involved in their development.

Conclusions

Experience with operator toll dialing shows clearly that it provides a marked improvement in toll service. This improvement will increase as progress is made toward the full application of the nation-wide automatic switching plan.

The development of long-distance dialing by customers is at an early stage. The results of recent trials, however, indicate that nation-wide customer dialing has service advantages and generally will be received with enthusiasm by telephone users. It is anticipated, therefore, that customer dialing will expand rapidly both on a regional and also on a nation-wide basis.

The service advantages of nation-wide automatic switching are not measured entirely by the increased speed and improved accuracy of connections. An important factor is the continued ability of the telephone system to meet the rapidly increasing demand for telephone service without making excessive demands on the available supply of labor. The development of local dial operation was absolutely necessary to handle the great growth of local telephoning which has taken place. Today, in many places, requirements for people for toll operations are very heavy and an increased amount of automatic toll switching is becoming more and more necessary to make possible handling the rapidly increasing number of long-distance telephone messages.

With this development there has been a marked increase of employment, The Bell companies today employ 244,000 operators compared with 131,000 in 1941. They also have employed many people to build and install about $300,000,000 worth of toll dialing equipment, to construct places to house it, maintain it, and carry out operating rearrangements.

With respect to the future, even with the nation-wide automatic switching plan in full operation and the local central offices arranged to permit customer dialing, there still will be a large amount of work for operators. They will be required to handle information and assistance traffic, person-to-person calls, collect calls, and other classes of calls which do not lend themselves to customer handling, as well as any individual calls which the customers may not wish to dial themselves.

The Bell companies necessarily have taken the lead in planning and applying these new developments. The plans, however, are all laid in such a way as to include telephone users in independent telephone company offices. The independent companies are being kept fully informed of these plans as they develop and are participating, as the development of their own plant makes it practicable and desirable, in extending the benefits of the new forms of operation to their own customers.

This long-term development has required the very close co-operation of all parts of the Bell system—American Telephone and Telegraph Company General Department, Bell Telephone Laboratories, Western Electric Company, Long Lines, and all of the Bell operating companies. Each installation of equipment and circuits and each operation is a part of a nation-wide system and must be closely co-ordinated. The close interrelation and working together of the various parts of the Bell Telephone System, research and development, manufacturing, engineering, and operating are necessary for the effective planning and execution of this tremendous project.

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No Discussion

Fundamental Plans for Toll Telephone Plant

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THIS paper covers the general switching plan and fundamental plant layout proposed for handling telephone toll messages throughout the United States and Canada using automatic toll switching.

There has been rapid growth in the number of telephones and in the volume of toll traffic, particularly long haul. Toll facilities are provided under fundamental plans, an essential part of which is a toll switching plan for setting up connections quickly between any two telephones. The introduction of mechanical operation and the general improvement in the transmission performance of the communication plant over a period of years make the introduction of certain modifications in the fundamental plans possible and advantageous at this time. The important new features and the service improvements which are provided by the proposed plans are outlined in this paper. The principal types and characteristics of circuit facilities available for use in the intertoll network are also described.

General Aspects of Toll Switching Problems

Switching plans providing for the systematic routing of toll telephone traffic have been employed by the communication industry for many years. These plans have contributed directly to the high quality of long-distance telephone service enjoyed by the public in the United States and Canada. This generally excellent service is the result of the co-operative work of many organizations including the Bell operating companies, many independent connecting companies and others in the United States as well as in adjoining countries. The techniques employed today reflect a great amount of research and engineering and improvements in manufacturing skill and in construction, maintenance, and operating methods developed over a period of many years.

Throughout the United States and Canada there are approximately 20,000 different places—cities, towns, and villages—that serve as toll connecting points. The telephone offices in each of these places have access through the toll network to practically all of the 50,000,000 telephones in the United States and Canada and also to most of the telephones in the rest of the world. Currently, the Bell operating companies are handling toll calls at an average rate of over 7,000,000 during a business day. The many millions of different connection possibilities which this number of calls involves require a definite and comprehensive switching plan.

Whenever practicable and economical, direct circuits are used to handle toll message traffic between two given points. Much of the traffic in the country is handled this way. However, a substantial volume of business, about 20 per cent, is handled as a matter of economy, by switching toll circuits together. Although the volume of traffic between different points may vary over a wide range, it is nevertheless important that adequate service be provided for all possible connections. For example, there are about 110 circuits from Chicago terminating in the toll office serving Minneapolis and St. Paul. These handle about 5,500 calls per day. On the other hand, only a few calls a year may be involved between some point in Western Minnesota and a point in Florida. The switching plan described in this paper is devised for the purpose of efficiently and effectively establishing connections between any two points regardless of their separation and regardless of whether traffic volume be a few calls per year or many calls per hour.

Elements of the Problem

In order to illustrate the problem, a specific example may be useful. Figure 1 is a map of Wisconsin and Minnesota on which nearly 1,200 circles indicate points at which exchange facilities may be connected to the toll network. The extent of the coverage in this area is typical of that found throughout the country.

The 150-odd larger circles represent existing offices known as toll centers, that is, places where operators record toll calls and perform other operations necessary to establish toll connections. These places have switching arrangements of various types depending on how they fit into the switching plan. Some may operate as control switching points in the nationwide plan as described later.

More than 1,000 smaller circles on the map represent tributaries, that is, towns where little or no toll operating is done. Toll connections to and from these points are completed at the toll centers which in general do the toll operating required.

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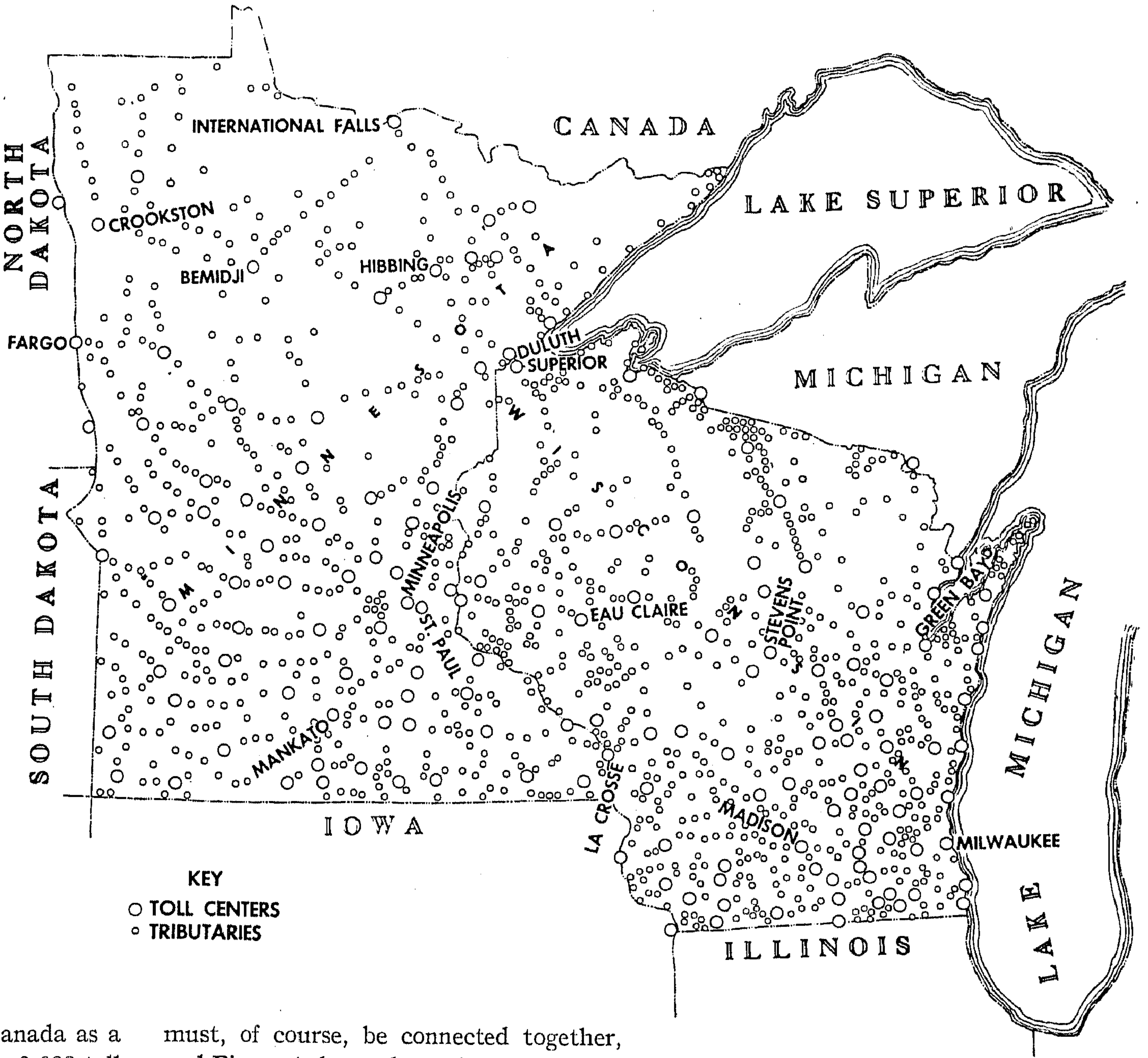


Figure 1. Toll centers and tributaries in Minnesota and Wisconsin

In the United States and Canada as a whole, there are approximately 2,600 toll centers. The remainder of the toll connecting points—about 17,500—are tributaries.

Figure 2 gives an idea of the variety and complexity of the network of circuit groups required to interconnect the toll centers in one area. Here each line represents a group of circuits known as intertoll trunks between two toll centers. Each group may contain anywhere from one to several dozen trunks. The location of the lines on the map is unrelated to the geographical routing of the trunks, and only a part of the circuit groups are shown. To get a complete picture, one should visualize that a cluster of relatively short circuit groups radiates from each toll center to its tributaries, of which there may be up to 15 or more.

Physically, the plant consists of a network of open wire lines, cables, and radio systems. On these, voice frequency or carrier operation is employed in each section, as required, to provide the necessary intertoll trunks. The routes of the lines in Minnesota and Wisconsin are shown by Figure 3. In this area there are no radio routes carrying telephone circuits, but a radio system between Chicago and Minneapolis is in the planning stage.

Areas like Wisconsin and Minnesota must, of course, be connected together, and Figure 4 shows the major Bell system toll routes that accomplish this. On a map of this kind it is not possible to include anything like the detail shown in Figure 3. One must visualize, therefore, that each state contains a network of routes generally comparable to those shown for Wisconsin and Minnesota.

This then represents the interconnection problem to be met by an orderly switching plan that will provide efficient, reliable, and fast toll telephone service between any two points.

Earlier Toll Switching Plans

Very early in the telephone industry it became evident that:

1. There must be a plan for connecting circuits together.
2. Switching centers with suitable equipment must be established in accordance with this plan.
3. Trunks must be provided in adequate numbers to connect every place to one or more switching centers and to interconnect the switching centers.
4. All this must be done in a way that makes it possible to provide good service at reasonable cost.

As time went on, early plans crystalized into what became known as the General Toll Switching Plan. A paper by Dr. H. S. Osborne outlined the principles of this comprehensive plan for handling telephone toll traffic in the United States and Eastern Canada.[1] It involved two classes of major switching centers, regional centers and primary outlets, and some classes of less important centers. It also set up methods of designing toll trunks to give adequate transmission efficiency on all possible toll connections. In use for the last 2 decades, this basic plan has been of great value in accommodating the tremendous growth of telephone toll business during this period.

Switching Plan for Nation-wide Toll Dialing

The earlier general switching plan was based on manual switching and on a toll plant made up, for the most part, of voice frequency circuits. The probability of operating irregularities and delays increases with the number of manual switches in tandem. Likewise, the transmission problem of operating many voice frequency trunks in tandem was so formidable that the number of intertoll

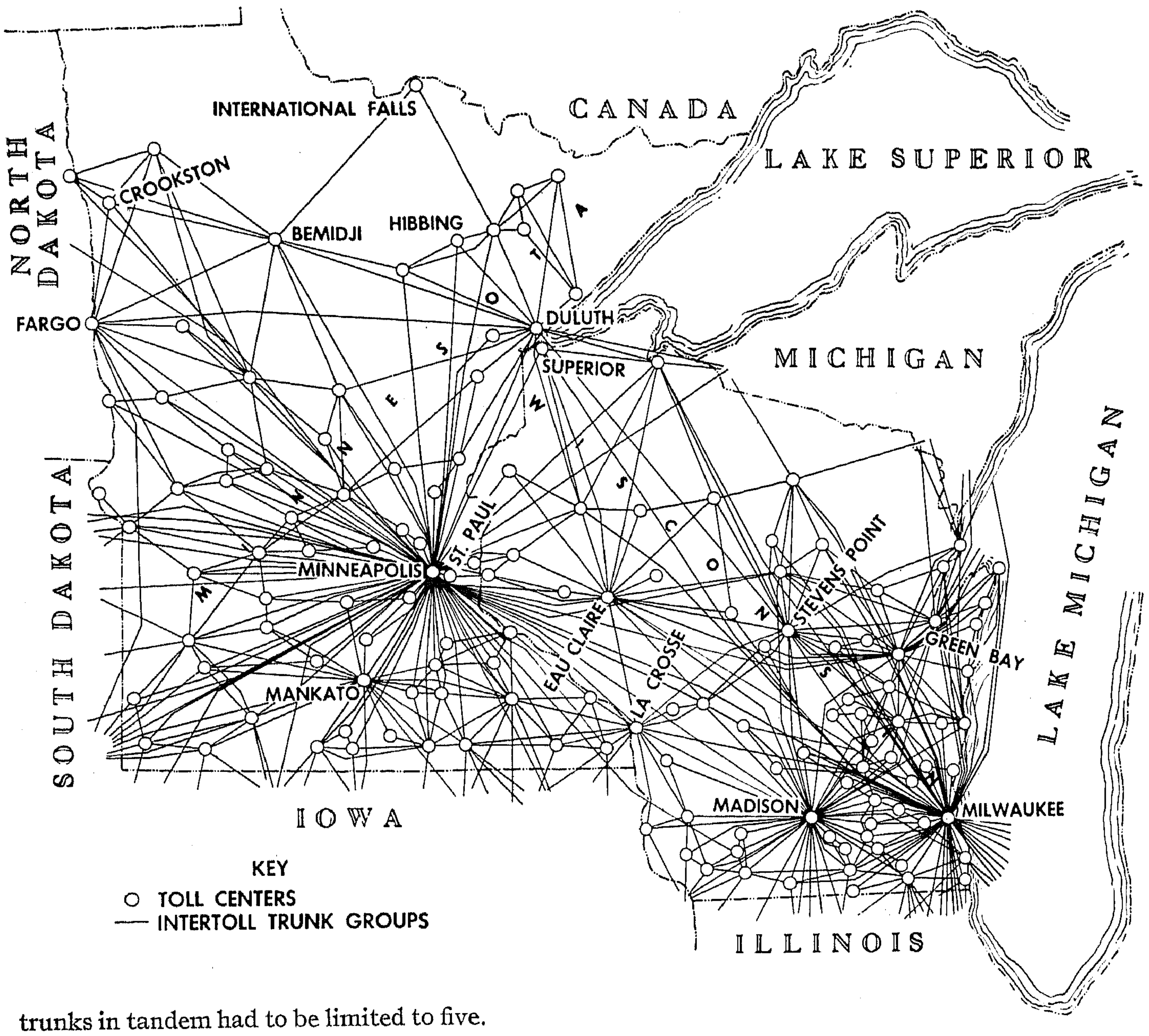


Figure 2. Principal intertoll trunk groups in Minnesota and Wisconsin

trunks in tandem had to be limited to five. In practice, switching was avoided wherever practicable and economical.

Impact of Mechanization and Improved Transmission Facilities

On the other hand, mechanical switching is very fast and is designed to be practically free of operating irregularities. Delays can be minimized by fast switching to alternate routes. Also, in the last 2 decades the use of carrier has grown from a relatively minor place in the toll plant to the point where it is now commonplace.[2] Carrier provides superior transmission performance. Limitations on switching are thus greatly reduced and economies are achieved under many conditions.

In addition, mechanization of local switching systems has proceeded rapidly. With mechanized toll switching, it is becoming possible to establish many toll connections with only a single toll operator and in some cases by customer dialing, without the assistance of any operator.[3,4]

Along with these developments has come tremendous growth in traffic. Since 1930 toll messages in the Bell operating companies and the Bell Telephone Company of Canada have more than trebled, growing from an annual volume of about 650,000,000 to about 2,000,000,000. Intertoll trunks over 25 miles in length have increased in number from about 28,000 to about 100,000. This continuing growth in traffic volume has required a large-scale development of plant facilities and has permitted a more extensive use of carrier than would have been practicable with a slower rate of growth.

Consideration of these factors which offer an opportunity to improve service has led to the gradual reorientation of the fundamental plans for the intertoll trunk plant which is now under way.

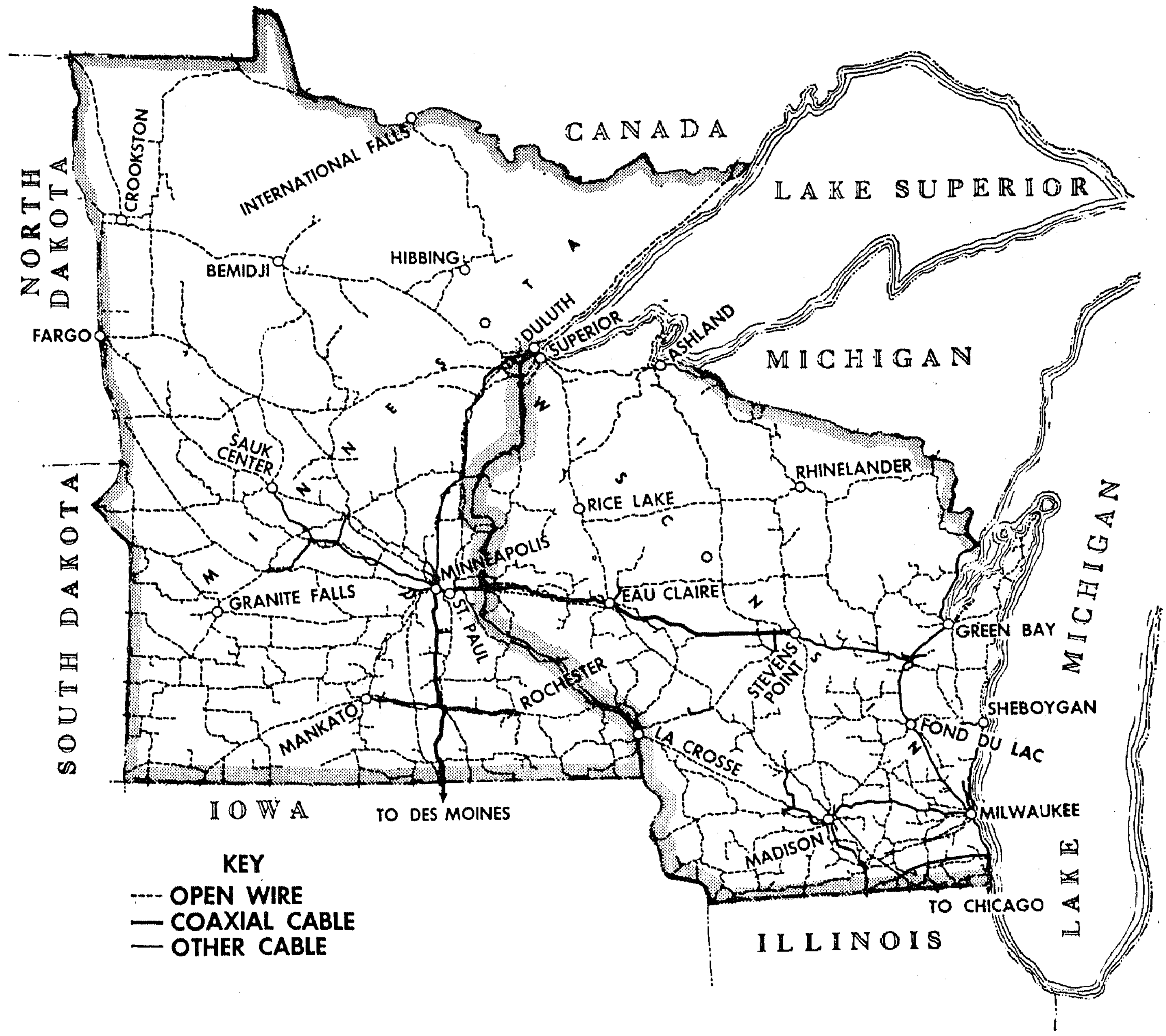
The New General Toll Switching Plan

Mechanization of switching and the use of improved transmission instrumentalities permits the design of the switching plant to be controlled primarily by the balance between the costs of transmission facilities and of switching facilities.

The new general toll switching plan contemplates as many as eight intertoll trunks in tandem on the most complex connections to be established. These eight trunks can be interconnected at switching points as described later. The plan further contemplates that wherever possible, the traffic will by-pass intermediate switching points. The number of switches that can be avoided depends on the volume of traffic between the two points concerned and on the traffic load at the time the connection is established.

The proposed plan provides a systematic grouping of switching points. Under this arrangement, each ordinary toll center (TC) serves a cluster of near-by-tributary points and has trunks to a home primary outlet (PO) which serves a cluster of toll centers. In some cases it appears practicable to utilize a simplified switching system at a PO, and in order to distinguish this type of center it has been designated a tandem outlet (TO). In turn, each PO or TO has trunks to a home sectional center (SC) which serves a section of the country varying in size from part of a state to all of several states, depending on the density of the population. Similarly, the United States and Canada are divided into nine regions, each having a regional center (RC) serving as a central switching point for all sectional centers in the region. One of these RC's (St. Louis) is termed the national center (NC). All of the higher orders of switching centers also act in the capacity of each of the lower centers. For example, any specific SC also acts as a PO and as a TC.

Figure 3. Bell System telephone toll routes in Minnesota and Wisconsin



This arrangement is illustrated in Figure 5 which covers approximately the same area as Figure 1 portraying the toll connecting points. Hibbing, Minnesota is shown as a representative toll center with the tributaries it serves. It is in the service area of the Duluth tandem outlet, the approximate boundaries of which are indicated. Duluth lies in the Minneapolis section, which includes a large portion of Minnesota, and is in turn one segment of the Chicago region which serves a little larger area than shown by Figure 5.

Under this arrangement, toll calls between two tributaries in the Hibbing toll center area can be completed by switching at the toll center. In a similar manner, any two points within the Duluth tandem outlet area can be served by switching at Duluth. The same treatment also applies for connections between any two points in the same sectional center area or in the same regional center area. For example, a connection from Hibbing to any point within the Chicago region (which involves more than six states as shown in Figure 7) requires no more intertoll links than Hibbing to Duluth, Duluth to Minneapolis, and Minneapolis to Chicago, and a corresponding number of links on through to another sectional center, and primary or tandem outlet to the toll center destination. Circuits between the toll center and tributaries are not referred to as intertoll trunks or lines but are classed as toll connecting trunks.

Where the volume of traffic warrants, direct circuits may be provided to by-pass the intermediate switching points included in the preceding example. Once such direct-circuit groups have been established, it is economical and advantageous from a switching standpoint to take advantage of their existence, using routes that involve a minimum number of switches. The basic routing plan is used when the more direct-circuit combinations are busy.

These routing arrangements contemplate the application of "high usage" and "final" trunk groups as an integral part of the plan. The high usage groups are direct groups which by-pass the higher order switching points wherever the routing of the call permits. These high usage groups can be engineered to carry high loads per circuit, with an adequate number of circuits in the final groups to take care of practically all overflows from the high usage groups during the heavy traffic periods. The high usage and final groups which could be used for routing calls between Hibbing, Minnesota and Davenport, Iowa are shown by Figure 5.

Generalization of the Toll Switching Plan

The generalization of the arrangements discussed for the Chicago region is illustrated in Figure 6. This shows diagrammatically all types of switching points in two regions and also indicates the relative position occupied by the national center in the switching plan. On this chart, the solid lines represent the final groups of trunks and the dotted lines represent high usage trunks. Examination of this chart will indicate that the mechanical switching system need perform only relatively simple toll switching operations at the toll centers. At other points the system must attempt to complete the call over the most favorable routes, in planned sequence, until the final route is selected.

For example, from a given primary outlet such as PO1 on a call destined for a toll center in the other region such as TC2, the switching equipment would attempt to complete the call in sequence over the routes marked 1 to 6.

Should Route 6, which is the final route, be selected because all of the trunks in the high usage groups marked 1 to 5 were busy at the time, the switching equipment at the SC would in turn try routes marked

A, B, C, and so forth, in attempting to complete the call. A fairly complete pattern of circuit groups is indicated in this illustration. Depending on the relative locations of the points concerned and the traffic load requirements, certain of the high usage groups shown may not exist. It is expected, however, that most TC's will have high usage groups to points other than their home PO's. Also each PO can be expected to have high usage groups to sectional centers other than its home SC. All regional centers will be interconnected with direct trunks, regardless of geographical location.

Control Switching Points

Because of rapid and complex switching operations required by the automatic equipment at PO's and higher order switching points (SC's, RC's, and the NC), these switching centers are called control switching points (CSP's).

As covered by a companion paper,[5] the switching equipment required at the CSP's is quite complex. This equipment must have a high degree of built in capability to perform quickly the circuit selection work associated with the alternate routing features of the switching plan. In addition, to help provide the transmission margins needed for satisfactory operation of the plan as contemplated, it must be arranged to connect circuits on a 4-wire basis rather than on a 2-wire basis, the latter being the arrangement used at most toll centers. The switching equipment at a CSP must not only provide for connecting one toll circuit to another; it must also perform the very important function of tying the toll networks which serve limited local areas together so that collectively they work as a smoothly functioning nationwide system. This becomes practicable when there is co-ordination between the design of the individual limited networks and the design of the over-all system.

The location of control switching points indicated by the nation-wide plan is shown in Figure 7. This also indicates the home switching center of higher order associated with each switching point. As the number of CSP's increases, the cost of the toll circuit plant decreases because each CSP can then be located closer to the cluster of ordinary toll centers which it serves. However, because of the cost of the CSP equipment, it is necessary to weigh the cost of circuit facilities with the equipment costs in a way that will result in the minimum over-all cost. Certain of the smaller primary outlets are being studied with the view of reclassifying them as tandem outlets (TO's). A tandem outlet occupies the same relative position in the switching plan as a primary outlet but is not a control switching point. The switching equipment employed is less complex than that used at control switching points and, therefore, provides for only limited alternate routing and does not have all of the advantages of 4-wire transmission.

Effects of Customer and Operator Toll Dialing

Customer dialing of short-haul toll calls has been in use, particularly in metropolitan areas, for some years. A trial of long-haul customer dialing over the intertoll trunk network and through the switching equipment provided for operator toll dialing was instituted at Englewood, N. J., in the fall of 1951. The local equipment includes automatic message accounting and permits Englewood customers to dial directly to about 11,000,000 telephones in ten metropolitan areas across the country. A trial installation of customer toll dialing, utilizing automatic message accounting equipment on a centralized basis rather than at each local office, is planned for Washington, D.C. in the fall of 1953. Initially customers will dial toll calls within the Washington metropolitan area and to such points as Baltimore and Annapolis. The favorable results and general acceptance of the trial at Englewood indicate extensive application of customer dialing of toll calls as conditions warrant.

The general introduction of customer toll dialing as this becomes desirable will affect the number and location of ordinary toll centers since calls handled by operators may be limited to assistance calls and to person-to-person, collect, and others which cannot be customer dialed. Indications are that toll operation for a number of smaller centers can be combined as the local service is converted to

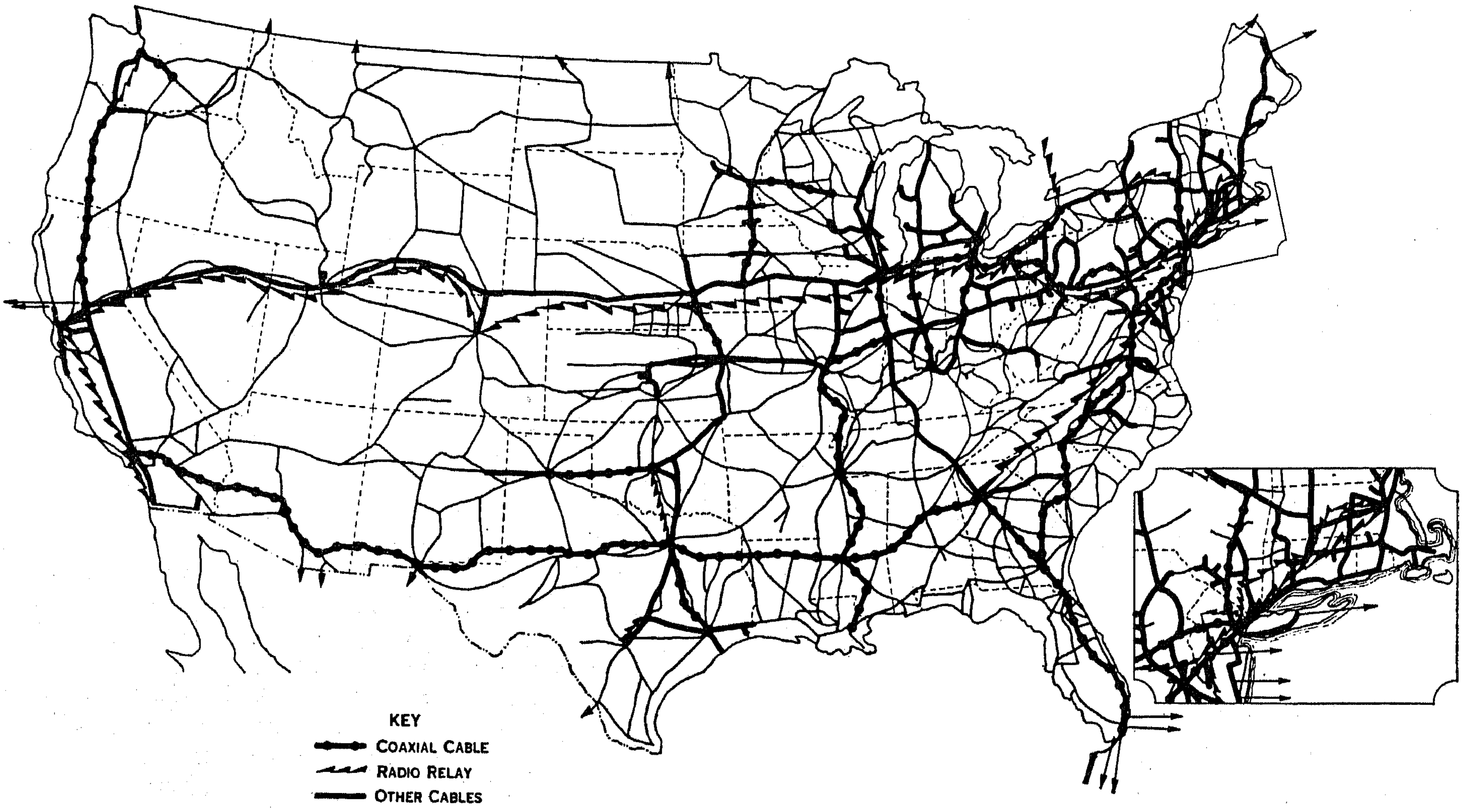


Figure 4. Principal toll routes of the Bell System

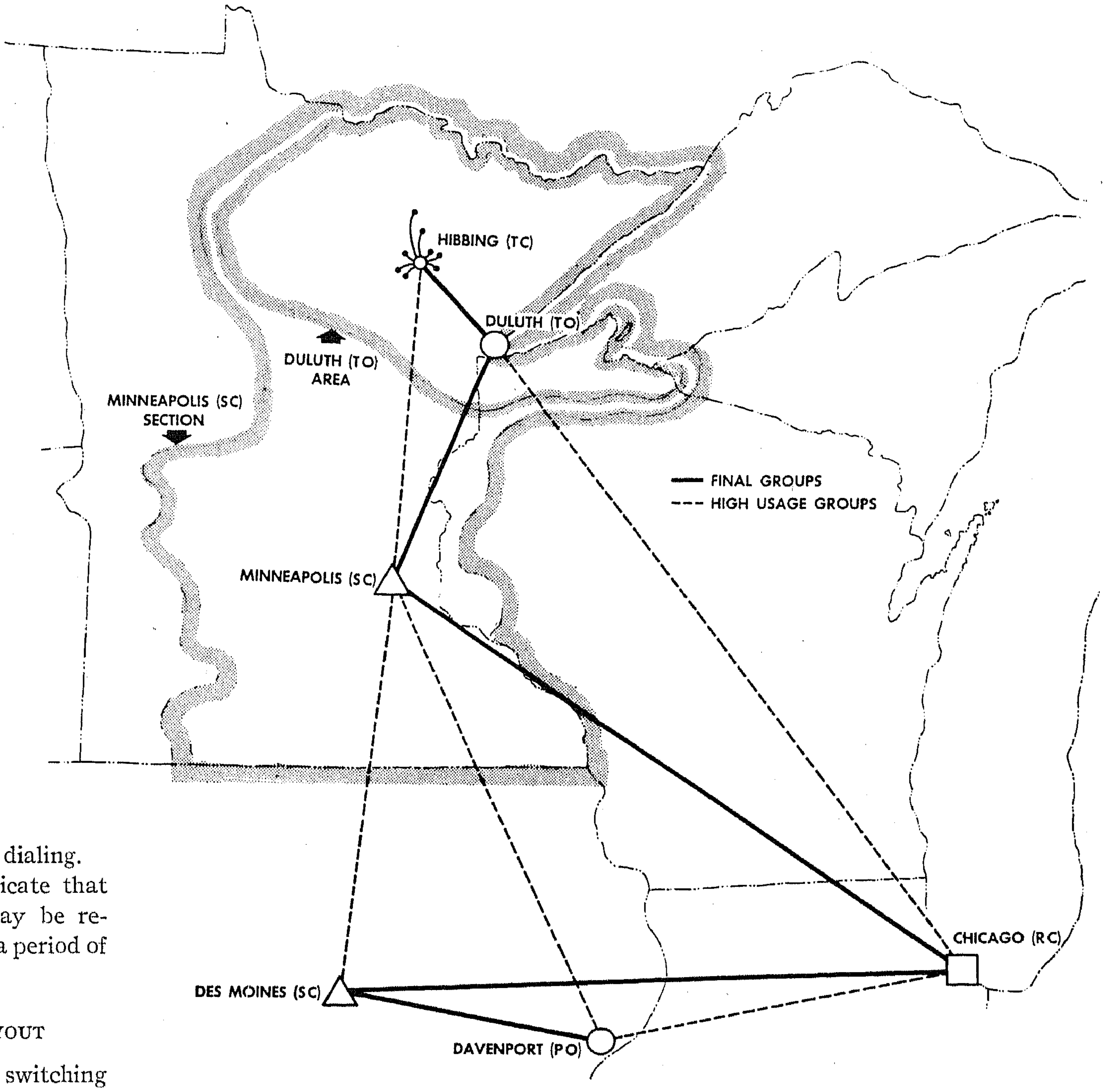


Figure 5. Intertoll trunks between Davenport, Iowa, and Hibbing, Minnesota, showing alternate routing possibilities

dial operation with operator toll dialing.

Studies now in progress indicate that the number of toll centers may be reduced by one-half or more over a period of years in many areas.

REACTIONS ON TOLL PLANT LAYOUT

The expanded general toll switching plan for nation-wide dialing, using a nation-wide numbering plan,[6] contemplates a degree of alternate routing far in excess of that used with the former switching plan designed for manual operation. This change along with the reduction in toll centers will have a marked effect on the normal flow of many traffic items through the intertoll network. As a result the arrangement of the present intertoll trunks will be significantly modified both in number, routing, and terminating points. It is necessary to take these facts into account in engineering toll plant additions so that they will lead toward an advantageous layout for future nation-wide dialing as well as meet the needs of the more immediate future. Fortunately, the effect is in the direction of greater concentration of circuits in main routes so that with the new cable and radio facilities available, over-all economy and better service should result.

Types of Transmission Facilities Used and Included in Switching Plan

The domestic toll network is an outgrowth of the demands of the business and the advance in communication technique over many years. At present, about 100,000 intertoll trunks over 25 miles in length and many thousand shorter toll trunks are in service throughout the country. They are provided generally by voice frequency or carrier frequency facilities. The choice of transmission facility on a given route is dependent on a number of factors, such as cost, length of haul, number of trunks in the cross section, numbers of trunks to be terminated at intermediate points, the types of terrain to be traversed, storm and other conditions affecting service continuity, and the transmission requirements of the circuits to be provided.

Voice frequency facilities equipped with repeaters as required are used on both open-wire lines and cables. At voice frequencies it is customary to derive three trunks known as a phantom group, from two pairs of open wires or from one "quad" (2 pairs) of loaded cable conductors. In general, the use of voice frequency facilities is now limited to shorter circuits.

Considerations of economy and service improvement led to the introduction of carrier operation into all types of toll plant as rapidly as the state of the art permitted. This directly affects the toll switching plan from the standpoint of routing and location of switching centers.

At present, carrier systems use four broad categories of facilities: open wire, conventional paired or quadded cables, coaxial cable, and radio.

Several types of open-wire carrier systems permitting from 1 to 15 telephone channels above the frequency band of the voice channel are now in use. In general, these systems are used where trunk cross sections are relatively small and where the terrain and weather conditions make open-wire lines economical.

Cable carrier systems presently permit the operation of up to 12 telephone channels on two pairs of cable conductors. These conductors may be in one cable or divided between two separate cables, depending on the type of carrier system, see Figure 8. Coaxial cable transmission systems currently provide up to 600 telephone channels per pair of coaxials, see Figure 9. A new coaxial system under

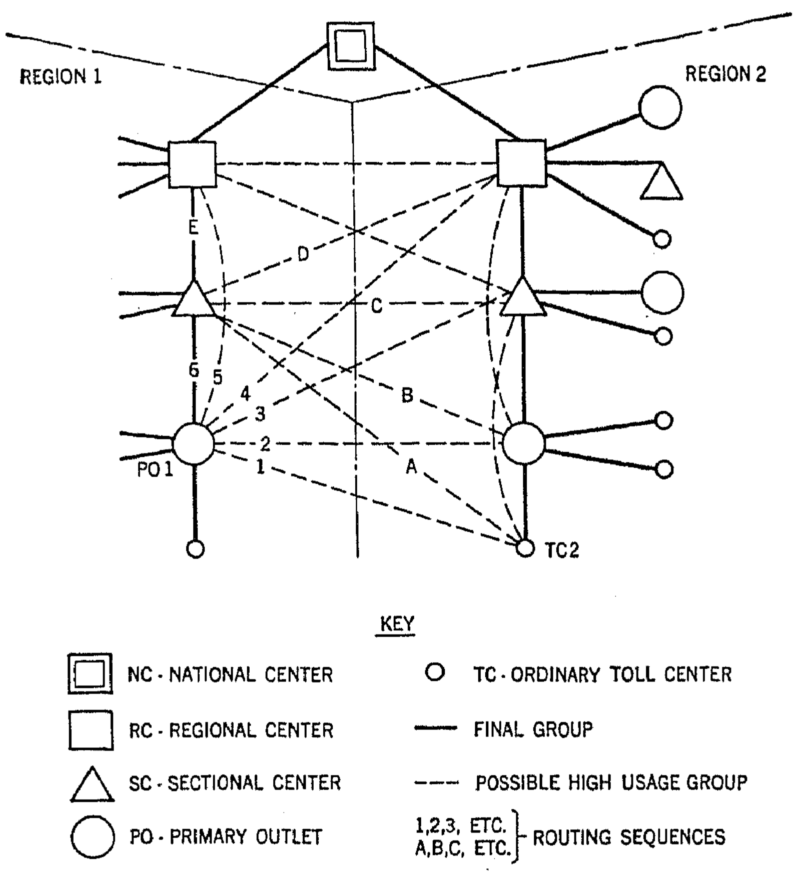


Figure 6. Illustration of intertoll routing pattern between two regions

development is expected to produce about 1,800 telephone channels per pair of coaxials.

Most of the applications of radio for toll telephone service now contemplated involve the use of point-to-point microwave systems. By employing channeling equipment at the terminals of these systems similar to that used for the present coaxial system, each pair of radio channels may provide up to 600 telephone channels. Several pairs of such radio channels may be operated through the same antennas, see Figure 10.

Radio systems are also useful in some cases where the number of toll trunks required is moderate, where diversity is desired, or where water or other natural barriers make the provision of wire circuits difficult or impracticable.

The type of facility to be used on a particular route is sometimes affected by requirements for other services such as teletypewriter, television network facilities, program facilities, private lines, and other factors.

Trend to Carrier-Type Facilities and Advantages to Toll Switching Plan

About 70 per cent of the long-haul toll message mileage in Bell operating companies is provided on carrier-type facilities as contrasted with 7 per cent in 1930, see Figure 11.

From the transmission standpoint carrier facilities offer marked advantages. They are inherently of the 4-wire type which minimizes the number of possible singing and echo paths on a circuit. Also, the speeds of propagation over carrier systems are generally higher than over voice frequency systems thereby further minimizing the echo problem. These features are of great advantage in reducing limitations on circuit design and layouts of the general toll switching plan.

Signaling Systems

In addition to the ability to carry messages, intertoll trunks must be provided with suitable signaling facilities.[7,8] These must provide a means of: first, attracting the attention of the distant point, either

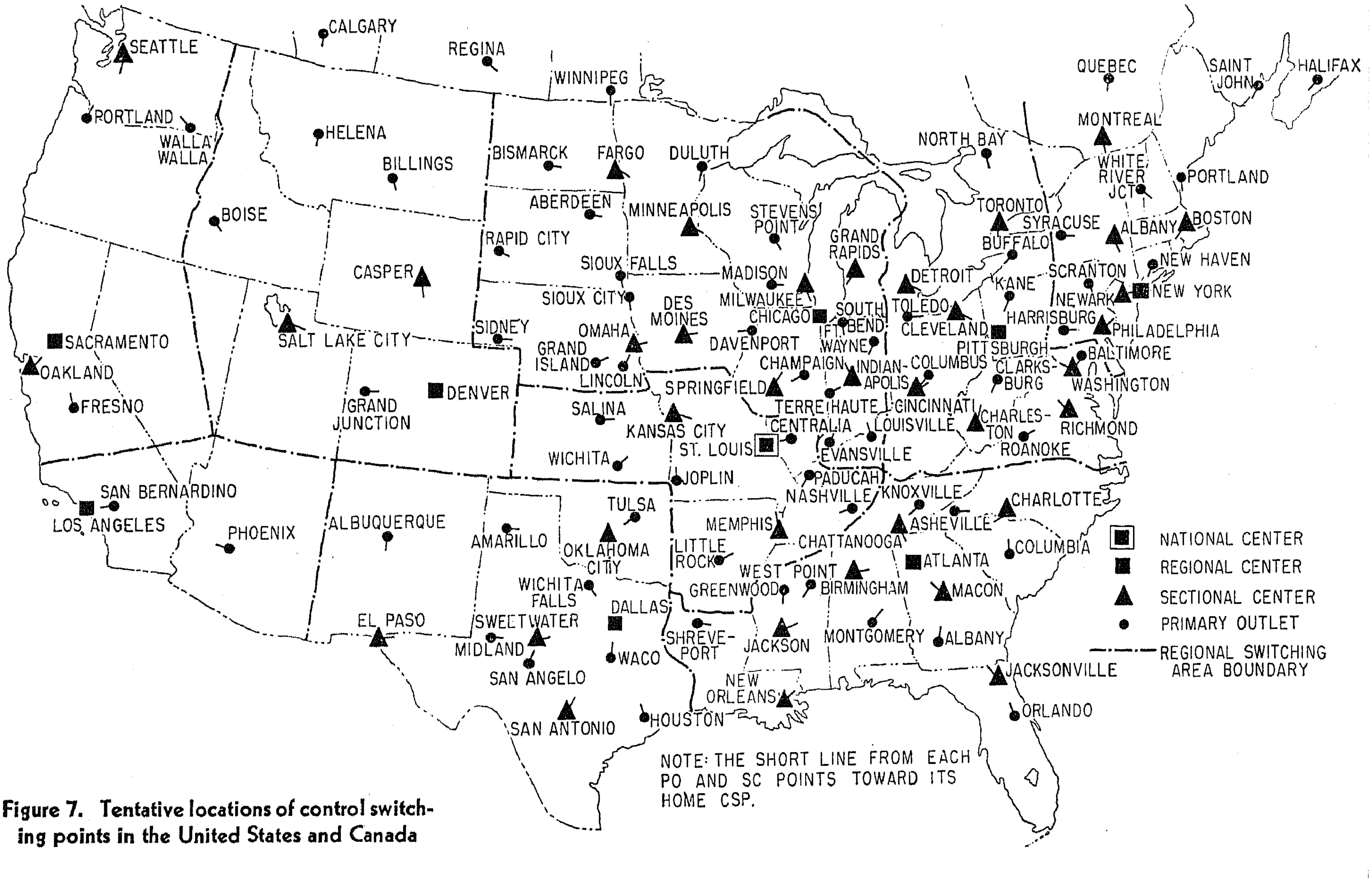


Figure 7. Tentative locations of control switching points in the United States and Canada

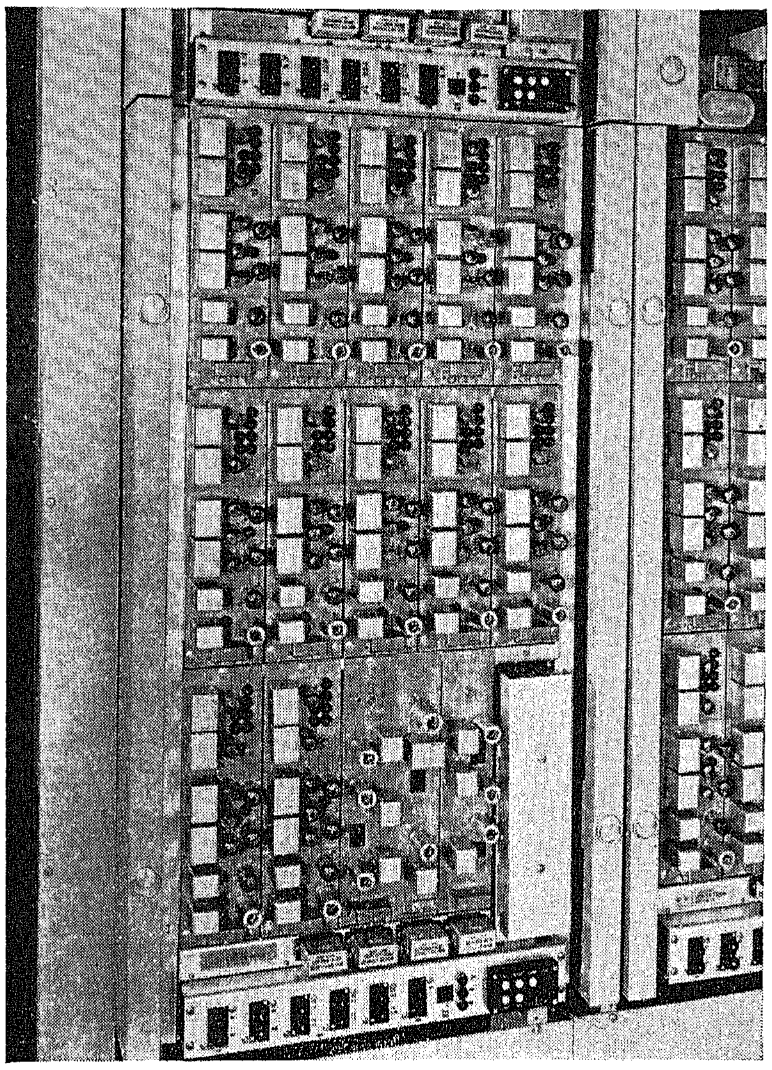


Figure 8 (left). Terminal equipment of type N1 cable carrier system. Provides 12 message channels with self-contained signaling equipment over two pairs of cable conductors in same sheath

Figure 10 (right). Microwave radio relay tower at Cotoctin Mountain, Maryland, on a New York-Washington radio route. Three hundred message circuits in service with more planned

an operator or automatic equipment, to the fact that a connection is to be established; second, in the case of dial operation, transmitting coded information in the form of pulses for establishing the connection; and third, transmitting a general class of supervisory signals including connect and disconnect signals, on and off switch hook signals, recall signals, and busy signals which are essential to the efficient operation of the switching plant. The circuit design contemplated in the over-all plan must take into account this requirement for transmitting signals as well as speech, to obtain accuracy and speed in setting up and taking down connections.

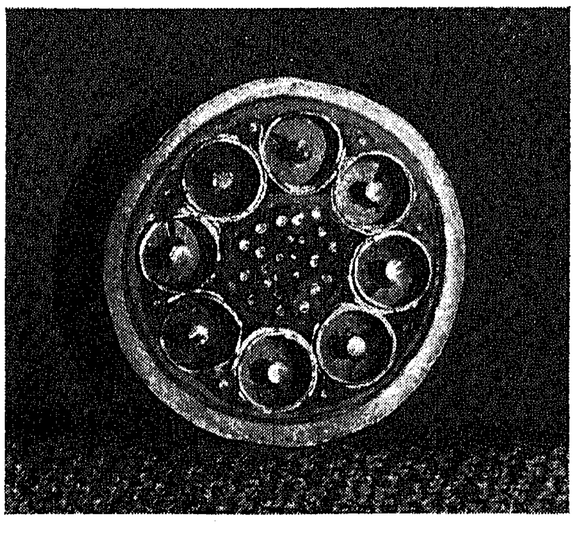


Figure 9. Coaxial cable. Cross section of cable containing four pairs of coaxials. Each pair can accommodate one 2-way coaxial carrier system

Transmission Design Aspects of Circuits for Nation-wide Toll Dialing

The more extensive use of alternate routing, together with the increase in maximum possible number of trunks in tandem associated with nation-wide toll dialing, tends to increase the problems of assuring adequate transmission of speech and signals on all possible connections. On the other hand, the use of 4-wire switching at important points and the definiteness of the routing patterns permit more effective use of the available facilities and thus tend to simplify the problem. Extensive studies indicate that, on the whole, the new toll switching plan will make feasible still further improvements in transmission. This is, of course, a desirable objective.

TRANSMISSION DESIGN OF TRUNKS

With dial operation, the number of trunks in tandem in a given toll connection may vary on successive calls. To avoid undesirable transmission contrasts and other adverse effects, it is important that every trunk be designed to operate as closely as possible to the theoretically correct transmission loss. The problem is complicated by the fact that the extent to which the echo, noise, and crosstalk will limit the performance of an individual link is not directly proportional to the length of the circuit. In fact, the minimum loss at which a particular circuit used singly or in various built-up combinations can theoretically be operated depends on the number, length, and characteristics of the other circuits connected in tandem with it. Arrangements for precisely adjusting the loss in the individual trunks for each call would be complicated. Adequate performance can be achieved, however, by compromise methods which provide for automatic adjustments in the loss of each trunk in accordance with:

1. When a trunk is switched to other intertoll trunks at both ends, it is operated at the minimum loss practicable. This loss is known as via net loss (VNL).
2. When the trunk is switched to another intertoll trunk at one end only, the loss is increased 2 decibels.
3. When the trunk is not switched to another intertoll trunk at either end a further loss of 2 decibels is added. This loss which is 4 decibels greater than the via net loss is known as terminal net loss (TNL).

The data and methods used in the der-

Table I. Comparison of Transmission Losses for Typical Intertoll Connections Realized under Manual Plan and Objectives under Dialing Plan

Number of Links in Intertoll Connection	Approximate Range of Losses Between Toll Centers in Decibels	
	Manual Plan	Dial Plan
1	4–12	4–8
2	8–14	5–12
5	9–20	6–13
8		7–13

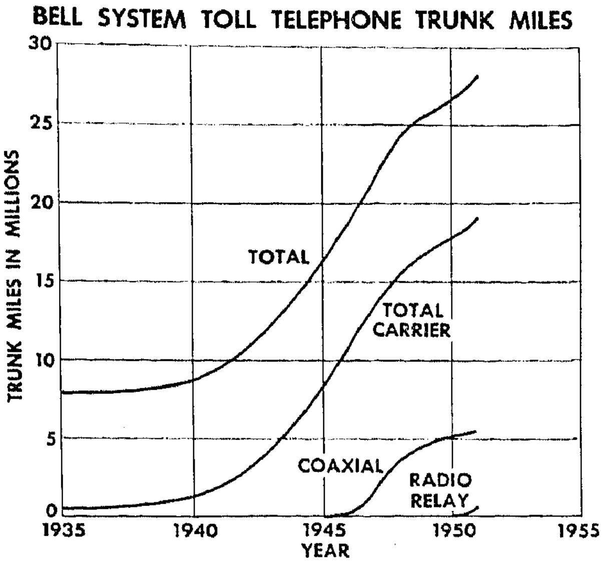


Figure 11 (left). Growth in Bell System intertoll trunk mileage showing trend toward more extensive use of carrier-type facilities

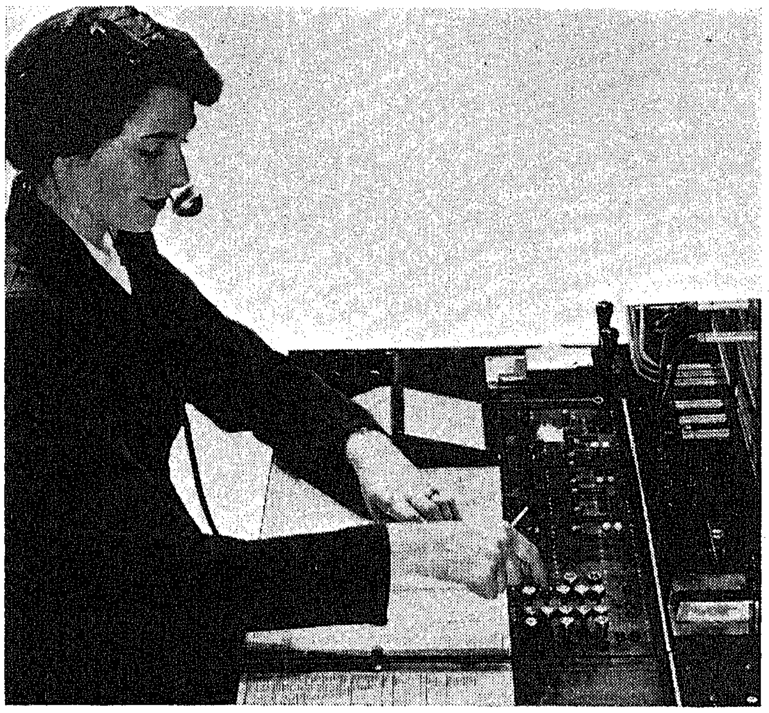


Figure 12 (right). Toll switchboard position with key set used for toll dialing

ivation of the via net loss are complex and not within the scope of this paper.

ASSIGNMENT OF FACILITIES AMONG TRUNKS

The definite routing patterns established for the toll machine switching operation impose more severe transmission conditions on certain classes of circuits than on others. For example, a trunk in a final group between a TC and a PO can become involved in an 8-link connection, whereas a trunk in a high usage group, say, between a PO and another PO will not be involved in more than a 3-link connection.

This creates a need and provides an opportunity for allocation of the available facilities among the various trunk groups in a way that will provide the best over-all service. For example, to the extent practicable it is desirable to assign carrier grade facilities to trunks in final groups that may be involved in connections with the maximum number of links. Facilities with less favorable transmission characristics may then be reserved for trunks groups that are used for connections in-ving fewer links.

ransmission Performance

Table I shows the approximate range of transmission losses between toll centers under the manual plan compared to ranges that appear practicable under the proposed fundamental plan, which, of course, permits more links in tandem.

TRUNK TRANSMISSION STABILITY

It is as important that the transmission loss of a trunk used in the contemplated toll dialing network be maintained at or close to its assigned value at all times as hat the assigned value be right. On muli-switched connections even a relatively mall consistent excess or deficiency in the oss in the individual trunks can accumuate to over-all excesses or deficiencies in oss large enough to cause difficulty by making it hard for people to hear if the attenuation becomes too great or by creating excessive echo, crosstalk, or noise if the loss becomes appreciably less than normal.

This subject has been extensively studied for the past several years and it appears that some changes in practices and the introduction of new methods of measuring results will lead to marked improvements. It is of some interest that one of the major factors in securing improvement appears to be the application of a statistical method of evaluating performance along somewhat the same lines as the quality control methods used in other fields of industry.

Since, with operator toll dialing only one operator is involved in many connections and with customer toll dialing there is no operator on the connection, it is extremely important that everything be right. This is typical of the requirements of any large scale push-button operation, see Figure 12.

Conclusion

The fundamental plans proposed for telephone toll switching provide a basis for the progressive mechanization of toll service. The installation of suitable switching mechanisms at control switching points and the provision of toll trunks utilizing the new instrumentalities will implement the toll switching plan. The plan is sufficiently flexible to adjust for changes in the telephone art as they develop. Also, the plan can fit in with the requirements of those companies whose plants connect with the Bell operating network should they desire to arrange for operator or customer toll dialing.$^{9-11}$

Average speed of service will be improved. The flexibility in plant design inherent in the new toll switching plan will increase service security and improve the utilization of the entire toll plant. In addition, adequate provision is made for the progressive introduction of customer toll dialing as this becomes practicable and desirable.

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No Discussion

Nation-wide Numbering Plan

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IN TELEPHONE language a numbering plan is exactly what the name implies, a plan or system of giving each telephone in a city, a town, or any geographical area an identity or designation which is different from that given every other telephone in this same area. This designation is the telephone number; it appears in the directory and in most cities on the telephone instrument itself. It is the address of the telephone in the telephone network. Just as it is essential for efficient postal and delivery service to have streets and house numbers clearly marked, it is important for good telephone service that the telephone numbering plan be such that it will be used with convenience and accuracy by the telephone customer.

A telephone number is comprised of two elements, a designation for the central office to which the telephone is connected and a number within the central office which identifies one particular telephone from all others served by that office. If there is only one central office in the city or town, the office designation is frequently omitted. A dial office is designed to serve up to 10,000 numbers with a limitation of four digits. Typical numbers are therefore MAin 2-1234, ADams-2345, 5-6789, and 3456, the office designations being MAin 2, ADams, and 5 with the last four digits in all cases representing the number within the central office.

There is a wide variation in the types of numbering arrangements in use today in the Bell System. This diversity arises from the fact that telephone communities vary greatly with respect to the number of telephones served, ranging all the way from New York City with its more than 3,000,000 telephones and 300 central offices to small villages and rural communities with perhaps a few score or a few hundred telephones.

In the 1920's when the Bell System embarked upon its program of converting local offices to dial operation, each exchange or city was in general an entity unto itself. Customers dialed local calls within their own city but all calls involving a toll or multiunit charge required handling by operators for timing and ticketing. There was no advantage, therefore, in making a numbering plan for a given city more comprehensive than required to serve the telephones and central offices in that city with a suitable allowance for the expected growth. Thus there were formed a multitude of local dial communities, large and small, within which customers could dial their own calls and connections between these telephone communities were established by operators.

Over the years these basic numbering plans which were established originally for local dialing have in many of the cities proved inadequate to furnish as many office codes as later events have shown are required. This is due to a variety of causes. The station growth in many places has outstripped all expectations and the number of central offices required to serve this unprecedented demand for service consume many more office codes than the original plans provided for.

In many places local service areas were changed so that customers could call into contiguous exchanges at local rates. To enable customers to dial into these nearby places the original numbering plans required expansion to include this increased number of offices. In addition, with the advance in the telephone art many cities introduced equipment for automatic charging on multiunit and short-haul toll calls so that customers could dial such calls directly instead of placing them with an operator for completion. In order to enable customers to dial these calls, it was necessary to expand the original city numbering plans to encompass wider and wider geographical areas.

In expanding the various types of numbering plans to serve a larger number of central offices than were originally anticipated, various expedients were resorted to. In the largest cities having 3-letter office codes a numeral was substituted for the third letter thus very materially increasing the code capacity from about 325 to about 500 and making it possible to form a number of codes using the same office name. The name CANal for example, instead of serving but one office may serve a number of offices, CAnal 2, CAnal 3, CAnal 4, and so forth. In the medium sized cities having 2-letter codes, expansion meant adding a digit to the code to all or in some cases to only a part of the offices in the city.

The 5-digit places usually were expanded by adding a digit to some of the numbers so that some of the telephones had five digits and others six digits in their numbers.

As a result of choosing originally a numbering plan which at the time seemed adequate and most suitable for the city involved and in many cases being forced to expand to meet changing needs, the Bell System now has a considerable variety of different numbering plans. These numbering plans, illustrated in Table I, all are adequate to serve the present local dialing needs for the cities in which they appear.

Having reviewed the numbering plan situation as it exists today in the various cities and towns, let us turn to the problem of handling toll calls. Under ringdown operation there is an operator at the outward toll center where the call originates and another operator at the terminating or inward toll center. On built-up toll connections there are additional operators at each intermediate toll switching point. The inward toll operators, who are familiar with the numbering plans in the offices served by their particular toll center, can be relied upon to connect to the desired station even though there is uncertainty on the part of the calling customer or the outward toll operator regarding the precise pronunciation or spelling of the name of the called office or the particular form of numbering system used at the called city.

Under operator toll dialing the inward operator is replaced by dial switching equipment under the control of the outward operator; hence the outward operator has no one to rely upon but herself in completing a toll connection to a distant city. With the present method the operator dials a code for each circuit group in the connection followed by the number of the called party which may consist of any number of digits from three to seven. The operator must refer to her position bulletin or to a routing operator for the correct circuit group codes unless she happens to remember them. Where the office to be reached has central office names, the operator must rely on routing information to determine how many letters of the name are to be dialed. The great variation in the number of digits to be dialed on different calls is a source of some difficulty and confusion to the operators.

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Table I. Numbering Plans in the Bell System

Place	Directory Listing	Customer Dials	Ordinarily Referred to as
Philadelphia, Pa.	LOcust 4-5678	LO 4-5678	Two-five
Los Angeles, Calif.	PArkway 2345 and REpublic 2-3456	PA 2345 and RE 2-3456	Combined two-four and two-five
Indianapolis, Ind.	MArket 6789	MA 6789	Two-four
El Paso, Tex.	PRospect 2-3456 and 5-5678	PR 2-3456 and 5-5678	Combined two-five and five digit
San Diego, Calif.	Franklin 9-2345 Franklin 6789	F 9-2345 F 6789	One letter, four and five digit
Des Moines, Iowa	4-1234 and 62-2345	4-1234 and 62-2345	Combined five and six digit
Binghamton, N. Y.	2-5678	2-5678	Five digit
Manchester, Conn.	5678 and 2-2345	5678 and 2-2345	Combined four and five digit
Winchester, Va.	3456	3456	Four digit
Ayer, Mass.	629 and 2345	629 and 2345	Combined three and four digit
Jamesport, N. Y.	325	325	Three digit

The present system of operator toll dialing by which operators use codes depending upon the routes to reach a desired destination, is a great improvement over the old manual handling methods. However, with the introduction of more modern toll switching facilities and the nationwide extension of toll dialing, it was realized that an improvement in the methods for dialing toll calls to distant cities was essential in order to realize the maximum speed and accuracy inherent in toll dialing.

These handicaps in the present toll dialing methods are to be overcome by establishing a nationwide numbering plan covering the United States and Canada by which each of the more than 20,000 central offices in the two countries is to be given a distinctive designation which identifies that particular office and that office only. This designation is to consist of two elements, a regional or area code and a central office code. Any outward toll operator, wherever located, will use that same designation in reaching that office through the dial toll switching network.

In a sense, all of the thousands of offices involved are to be treated as though they were contained in one huge multioffice city. Toll operators will use the area code and the office code in reaching an office situated outside her own numbering plan area, while on calls to points within her own numbering plan area she will dial only the number as listed for toll in the directory. In principle the method employed is to divide the two countries geographically into numbering plan areas and to give each of these areas a distinctive code, see Figure 1. Within each of these numbering plan areas each office will have a code unlike that of any other office in the same numbering plan area and also unlike any area code. Hence for toll-dialing purposes each office will have an area code and central office code which will form a combination unlike that of any other central office in the two countries.

In this geographical division into numbering plan areas, border lines between states and between Canadian provinces generally have been used as numbering area boundaries. Since about 500 central offices are the maximum number which can be served in a numbering plan area, it is necessary to divide the larger and more populous states and provinces into two or more areas making, of course, due allowance for growth. New York state with the largest number of central offices is divided into six numbering plan areas; Pennsylvania, Illinois, Texas, and California have four areas each. Other divided states have three or two areas depending upon the number of offices to be served. Approximately 90 areas are being provided, with 14 states and two provinces served by two or more numbering plan areas, the remaining states and provinces by one area each.

In fixing the intrastate numbering plan area boundaries of subdivided states, among other considerations effort was made to avoid cutting across heavy toll traffic routes in order to have as much of the toll traffic as possible terminating in the area in which it originated. The advantage of arranging the numbering plan areas in this manner is readily apparent since on this traffic which does not pass an area boundary the area code is not required.

Let us now consider the composition of the area codes. As indicated previously they must be of a type which will enable the switching equipment to distinguish them from the codes of central offices.

On the telephone dial plate letters are assigned only to the dial positions 2 to 9, inclusive (on some dial plates a Z appears on the 0 position but the Z is never used in a central office code), hence any office code will always avoid a 1 or a 0 in the first two places. The digits 1 and 0 can, therefore, be used in area codes to distinguish these from office codes. It is not practical to use them as initial digits of area codes since customers dial 0 to reach operators and the local dial equipment is arranged to ignore an initial 1 for technical reasons. A 1 or 0 in the second place, however, can be employed in an area code without conflicting with any central office codes or interfering with any existing practices. Accordingly, the area codes will consist of three digits with either a 1 or a 0 as the middle digit, 516, 201, and so forth. A few codes of this type are now in use, leaving a practical total of 152 of these area codes available as compared to approximately 90 assigned to our present numbering plan areas. This will provide a comfortable spare for additional future numbering plan areas or possibly for reaching overseas points which may be incorporated later into the toll dialing network.

As shown on the chart, states and provinces such as Montana or Alberta which are contained in a single numbering plan area will have area codes with a 0 as the middle digit to distinguish them from areas in divided states such as Texas where the middle digit will be a 1. This is to enable toll operators to differentiate between the two classifications of areas. On calls to single area states the operators will always know that every call to the state in question uses the one area code, whereas on calls to subdivided states additional information will be required to determine which of the several area codes should be employed to reach the particular destination. It is proposed to show on the operator position bulletin the codes of all single area states and the codes of all frequently called cities in multiarea states. The area codes of the less frequently called places in the multiarea will be obtained from a routing operator.

Within each numbering plan area each of the 500 or fewer offices are to be given a 3-digit office code which will be different from that of any other office code in that same area. Ultimately each central office will be given a 2–5 type of number consisting of an office name and five numerical digits, such as LOcust 4-5678, illustrated for Philadelphia. In the larger cities customers will dial seven digits, LO 4-5678, on local calls to numbers in the same exchange. In many of the smaller places the customers on local calls will dial only the numerical digits, the office name being employed for toll dialing purposes only.

Considering the thousands of central

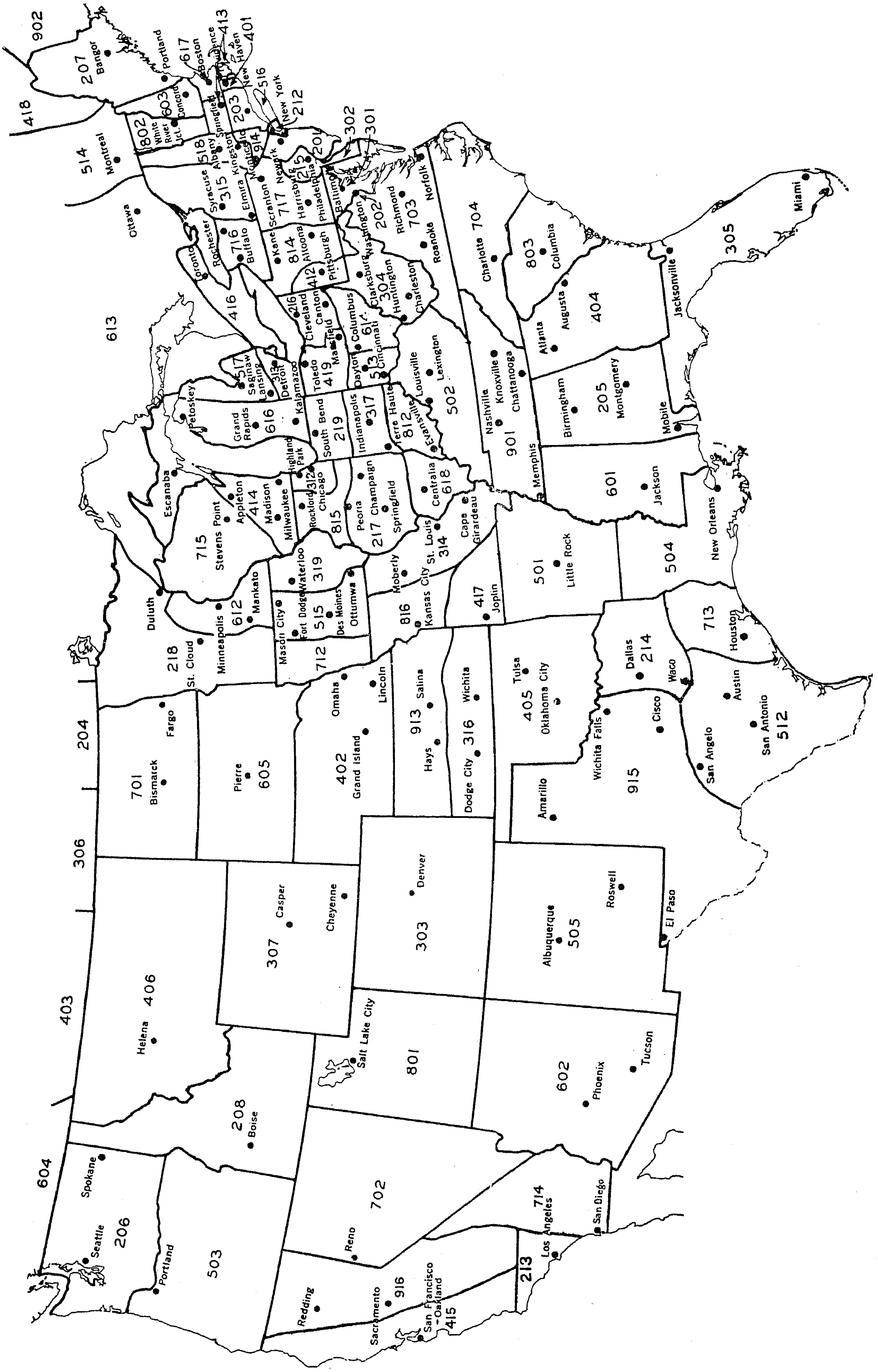

Figure 1. Numbering plan areas with codes

offices which now have numbers other than the 2–5 type and the fact that to change existing numbering systems is a difficult and often costly procedure, it will be a number of years before this ultimate objective is realized. As a practical measure, therefore, it will be necessary during this interim period, before the central office names with the 2–5 type of number are established everywhere, to employ for operator toll dialing office codes which in many cases may not be derived from the customers' telephone number.

In dialing to a combined 2–4 and 2–5 city, for example Los Angeles, the 3-digit office code for the Parkway office which has six digits in the local number, will be PAR, whereas to reach the Republic 2 office having seven digits in the local number, the office code will be RE2. To call a telephone in Winchester, Va., with only four digits in the local number, the operator will use a code consisting of numerical digits only, such as 294 which, of course, must be different from every other office code in this numbering plan area. To secure the particular office code to be used in reaching an office where the called number does not furnish complete information, the toll operator must refer to a position bulletin or the route operator. This reference work, of course, takes time and, therefore, imposes a delay in completing the call.

In addition to giving a distinctive 3-digit code to each office within each numbering plan area, each toll center also will be given a 3-digit code to enable outward operators to reach inward information, and delayed call operators at toll centers in distant cities. Calls to these operators will be routed in the same manner as calls to customers except that the operator codes will be used instead of a station number and a toll center code in place of a central office code.

The central office names now in use in the various cities in the System were chosen, generally speaking, on the basis of their suitability for customer dialing within the city itself. Many of these names are unfamiliar words to operators and customers in distant cities and the use of these names contributes materially to the operator dialing errors. This situation is gradually being corrected by using for new offices names from a System-approved list and replacing existing names which experience has shown to be particularly troublesome by names from this list.

While numbering plans are important in operator toll dialing, they play an even more essential part in the dialing of toll calls by customers. Operators can be trained to adapt their dialing procedures to the type of local numbering system encountered in the called city even though more time is consumed and more errors result than would be the case if all telephone numbers were of a uniform type. Customers, however, could not be expected to follow any plan which requires a variety of different procedures to be used in reaching different cities. Only a numbering system which is readily understandable and which customers find convenient to use and one which they can use with a very high degree of accuracy will suffice. The need for accuracy is readily apparent since with the customer's telephone being given access to the intertoll network without the intervention of an operator, a call which is misdialed can be routed to a telephone thousands of miles from the desired destination.

At present customer dialing of toll and multiunit calls is for the most part confined to situations where the call can be completed by the use of the number as listed in the directory without any additional digits being dialed. In a few cases as from Camden, N. J., to Philadelphia, and certain offices in Northern New Jersey to New York City, the code 11 is prefixed to the listed number. In the case of the current trial of customer toll dialing at Englewood, N. J., the customers are using area codes such as 415 for Oakland, California, 312 for Chicago, and so forth, dialing only into those cities which now have the 2–5 type of numbering.

From the Englewood experience it can be confidently predicted that this form of dialing, that is, an area code followed by a telephone number consisting of a uniform number of digits, is one that customers will use with a reasonable degree of convenience and accuracy. The problem, therefore, to meet the requirements for nationwide customer toll dialing, is to establish universally for all central offices regardless of size and location a uniform pattern of numbering for toll purposes. The only form of number completely filling the needs is the 2–5 system, which is that used in the largest cities today.

Accordingly, in order to implement the program for customer dialing of toll calls on a nationwide basis, it will be necessary to place all telephone numbers on a 2–5 basis with the code of each office different from that of every other office in the same numbering plan area. Thus each of the 50,000,000 telephones in the United States and Canada will have, for toll dialing purposes, a distinct identity consisting of ten digits; a 3-digit area code, an office code of two letters of an office name and a numeral, and four digits of the station number within the office. Typical numbers for toll dialing would, therefore, be 601-CA3-4567 or 317-MA7-6789. As with operator toll dialing, on a toll call which terminates in the same numbering plan area in which it originates, the area code will be omitted and the office code and station number—a total of seven digits will be used.

With this universal 2–5 type of number, local calls in and about the larger and medium sized exchanges will be completed by dialing the entire seven-digit number. For many of the smaller places in the more isolated sections, 5-digit or 4-digit dialing frequently will be employed where this number of digits will be adequate for all of the telephones in the customers' local dialing area. For these offices with five or four-digit local dialing and for offices in the larger places served by certain types of dial equipment, as they are arranged today, it will be necessary to prefix the dialing of toll calls by a transfer or directing code to permit the customer getting from the local office into the toll network.

Independent of the advantages of a universal 2–5 numbering plan for nationwide operator and customer toll dialing, the Bell System has made considerable progress in this direction over the past several years. New York and Northern New Jersey adopted 2–5 numbering in 1930 in order to take advantage of the flexibility of office code assignments and the large code capacity which this type of local numbering provides. Since World War II many cities and their environs such as Chicago, Boston, Philadelphia, San Francisco, Oakland, Pittsburgh, Milwaukee, Providence, and a number of smaller cities have followed suit. Presently about 12,000,000 telephones are in areas which have 2–5 numbering exclusively in addition to perhaps 2,000,000 telephones with 2–5 numbers in mixed 2–4 and 2–5 areas. Another 5,000,000 telephones are already planned for conversion to 2–5 numbers within the next several years.

The entire program will take many years to realize but it is one which must be accomplished in order to achieve the best results in operator toll dialing and make it possible for a customer at any telephone in the United States and Canada to reach a telephone anywhere in the two countries by dialing without the assistance of an operator.

No Discussion

Automatic Toll Switching Systems

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THIS paper deals primarily with the major switching centers required for the nationwide automatic switching plan. These are called control switching points (CSP's) and are supplied with switching equipment endowed with great versatility and a high order of mechanical intelligence. Mr. Pilliod's paper[1] explains how for purposes of circuit layout and routing, they are assigned different rankings as follows, starting with the lowest ranking: primary outlets (PO's), sectional centers (SC's), regional centers (RC's), and one national center (NC). Substantially the same equipment is to be provided for all of these centers so that they all will have inherently the same capabilities. They will, however, differ greatly in size. In the United States and Canada, as now envisaged, there will be somewhat under 100 of these CSP's.

The system which the Bell Telephone Laboratories developed for use at CSP's and which embodies all of the features required at those important switching points is based on the toll crossbar system[2] now in service and has been constructed by the addition of the necessary CSP features to the basic structure of that system.

Functions of the CSP Switching System

The system is designed to be suitable for location in either a step-by-step or a panel-crossbar local area. In addition to the functions required for operation as a CSP, it must, of course, perform the normal toll switching functions required of any system for switching the toll traffic characteristic of the locality it serves. These may be stated very briefly.

ORDINARY TOLL SWITCHING FUNCTIONS

It accepts calls either directly from operators or from senders in distant offices. In the interest of economy, it accommodates itself to the signaling language the operator's position or sender is equipped to deliver. Calls from operators may be either in the dial pulse (DP) or multifrequency[3] (MF) form. Calls from senders will be in the MF form. DP pulsing is the decimal type delivered directly by the dial and is at the rate of about one digit per second. MF pulsing represents a particular digit by a combination of two out of five frequencies in the voice range; it uses one of these frequencies in combination with a sixth frequency to produce a signal indicating the beginning of pulsing, and a different one of the five in combination with the sixth for an end of pulsing signal. It is transmitted from senders at the rate of about seven digits per second. Operators usually key at the rate of up to two per second.

The toll switching system completes calls to various types of mechanical toll and local offices and to operators, using the form of signaling dictated by economy for each call. For distant toll offices and local offices using step-by-step equipment DP will be transmitted; for other CSP's and usually for local crossbar offices, MF will be transmitted and at manual toll offices an operator will be called in either automatically on seizure of the toll line or by sending a ringing signal over the line but no pulses will be transmitted. Forms of pulsing different from either of these are used for local panel offices and for local manual offices in panel-crossbar areas.

It must transmit signals in one direction for initiating, holding, and releasing the connection and in the opposite direction to indicate to the originating end when the called subscriber answers and hangs up. These signals must be in a form suitable for propogation over the medium which carries them.

It must exercise control over the amount of amplification of voice currents introduced at the switching point so that a proper grade of transmission will be furnished.

All of these functions are performed by toll crossbar systems already in service. The features that distinguish the new system are those peculiarly characteristic of CSP operation.

CSP FUNCTIONS

The following features which will be built into the equipment at control switching points are commonly referred to as CSP features:

1. Storing and sending forward digits as needed.
2. Automatic alternate routing.
3. Code conversion.
4. Six-digit translation.

The first of these features is basically essential for implementation of the plan. The second produces faster service and important economies in outside plant. It also provides protection against complete interruption of service in case of failure of all circuits on particular routes. This aspect of the feature is so important that automatic alternate routing also may be considered essential. The other two features are provided for reasons of economy and produce economies of such magnitude that they are very much worth while.

Storing and Sending Forward Digits as Needed. The necessity of providing this feature in CSP switching systems arises from the nature of the numbering and switching plans. The numbering plan[4] is constructed with the objective of using a minimum number of digits to give each telephone user in the country a distinctive number.

Numbers delivered to the CSP equipment are in the form ABX-XXXX if the called place is in the same numbering area as the CSP. AB represents the first two letters of any office name and X represents any numeral. If the called place is in another numbering area this set of digits will be preceded by X0X or X1X. X0X or X1X is the area code, ABX the local office code, and these are the digits used for routing purposes. Regardless of the number of switches required to complete the call, these two sets of code digits are all that will be supplied. They are universal codes in that they identify specific destinations—any place in the United States or Canada—and for a particular destination the same set of digits will be used wherever the call may originate. All CSP's must, therefore, be able to advance a call toward the same place when the same set of digits is received.

To make use of destination codes possible, each CSP must store the digits as received and pass along to the next point whatever digits may be required there for advancing the call. If the next point is a CSP not in the home numbering area of the called place, the complete 10-digit number will be sent forward. If it is a CSP in the home numbering area of the called point, the area code will be dropped and the remaining seven digits will be sent

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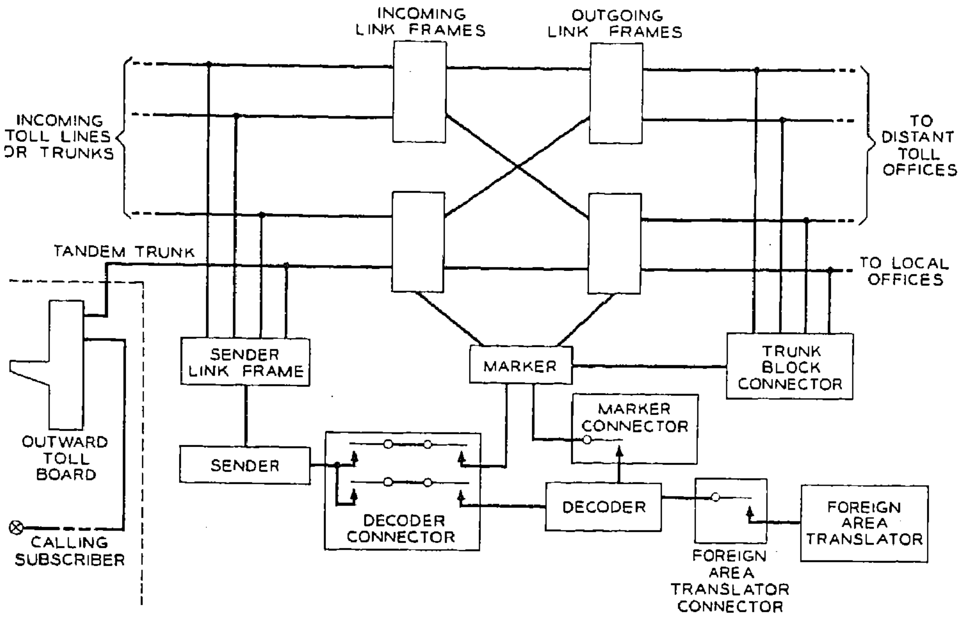


Figure 1. Schematic diagram of crossbar switching system for CSP's

forward. That CSP may in turn complete to a local office directly, dropping the office code, or through a step-by-step TO (tandem outlet) or TC (ordinary toll center), substituting arbitrary digits for the area or office code, thereby exercising the third of the listed CSP features.

Automatic Alternate Routing. The system is arranged to offer the maximum number of alternate routes possible under the switching plan. As explained by Mr. Pilliod[1] a maximum of five alternates actually will be used. This number is possible, of course, only at PO's since higher ranking CSP's have fewer CSP's above them in the final chain.

Code Conversion. This refers to the ability to substitute one, two, or three arbitrary digits for the area code, the office code, or both. It is economically important to be able to do this because it makes it possible to work with the step-by-step equipment extensively used in local offices and in toll offices in TC's or TO's without the changes in local numbering plans or rearrangements—and in some cases extra selectors—required for the step-by-step TC's or TO's to use ABX codes for routing purposes. Even though eventually all customers are listed as ABX-XXXX and TC's are arranged to use the listed number for routing the calls, this will not be accomplished for some time. Moreover, after such arrangements are in effect there will still be need for code conversion, particularly for routing calls through TO's. Many combinations of digit dropping and substitution are required to cover all possible cases.

Six-Digit Translation. When a CSP receives a 10-digit number, it is sometimes sufficient to translate only the area code digits and sometimes necessary to translate both the area and office codes. If all points in the called area are reached by the same route out of the particular CSP concerned, the area code will suffice for selection of the route. If some points are reached by one route and others by one or more different routes, the office code also must be translated to determine which route should be selected.

Basic Arrangement

In the CSP switching equipment talking connections are established through crossbar switches.[5] Incoming and outgoing toll lines and toll connecting trunks are terminated on crossbar switch frames with linkage between them to provide full access. The switches are controlled by equipment common to the office, each item of which is held only long enough to perform its task in setting up the connection.

The major items of common control equipment are senders, markers, decoders, and translators. The basic functions of the senders are the same as in other common control systems, that is, registering incoming digits and sending them out as directed. A departure from prior practice is made in the design of the marker. In other crossbar systems the marker is the principal seat of the mechanical brains. It not only controls the actual establishment of the connection but also does the translating to determine what connection should be established and what information should be passed to the sender for further disposition of the call. In this system, the marker still controls the actual setting up of the connection, but it acts on instructions received from the decoder where the major portion of built-in intelligence resides.

The decoder accepts code digits from the sender, translates them, makes selection of alternate routes, and gives instructions to markers and senders to enable them to carry out their assignments. To do the translating job, the decoder has one, and in some cases two, translators permanently associated with it and in addition has access to a common group of translators called foreign area translators which can be used by all decoders as required.

The relationship of the principal elements of the system to each other is depicted in the schematic diagram, Figure 1.

Method of Operation

The manner in which the various elements of the CSP system and the CSP systems at various locations co-operate to implement the nationwide switching plan may be understood best by following the progress of a call which demands the exercise of the characteristic CSP functions.

Assume an outward operator in Atlanta has received a station-to-station call for a subscriber in Monticello, Maine, whose number is ACademy 4-2345, that Monticello is a tributary of Houlton, a step-by-step TC, that Bangor is a step-by-step TO serving Houlton as its home TO, and that the circuit groups provided are as indicated in Figure 2. The dotted lines represent high usage groups and the solid lines final groups.

The Atlanta operator plugs into a tandem trunk to the toll crossbar system in Atlanta, thereby causing a sender to be attached to the trunk through the sender link frame. This causes a lamp signal to be displayed to the operator, indicating that she may key the number. She keys 207-AC4-2345 plus a start signal (signifying end of keying) into the sender and leaves the connection to handle other calls. She will give this call no further attention until the lamp associated with the cord circuit used in establishing it signifies either by flashing that the call was not completed due to a busy condition of the called line or to circuit congestion, or by going dark that the called subscriber has answered and she should start timing the call.

As soon as the area code 207 is received by the sender, it calls for a decoder and gives it the code. The decoder, by means of a self-contained translator, finds that the area code is sufficient for routing purposes, that the first choice route is by way of Boston, the second New York, and the final St. Louis, Without consulting other circuits it will know in which of these groups an idle circuit may be found. Let us assume that the circuits to Boston are all busy but that there are one or more idle circuits in the New York group. The decoder calls for a marker and tells it which group of leads to test, and also causes the sender to be connected to the particular marker it has selected.

The marker, following instructions from the decoder, is connected to the appropriate trunk block connector. This is one of a group of common circuits giving access to "blocks" of trunks for allowing the marker to locate an idle trunk. The marker examines the test leads of the individual toll lines to New York, and as soon as it has selected an idle circuit it so informs the decoder. The decoder then tells the sender to send all digits forward by MF and leaves the connection to accept another call. This information is relayed from the decoder to the sender by way of the marker. The work time of the decoder has been in the order of 1/2 second.

The marker determines the identity of the frames on which the incoming and outgoing circuits are located, finds an idle path between the two circuits, and sets up the connection. After checking the path through the switches to be sure that there are no troubles, it notifies the sender that its task has been completed and then leaves the connection. Its work time has also been in the order of 1/2 second.

In the meantime other digits have been coming in to the sender but it does not wait for all of them to arrive before advancing the call. When the marker selected the circuit to New York a signal was sent forward immediately to summon a sender in the New York switching system. The process of attaching the sender in New York was carried on concurrently with the establishment of the connection through the switches in Atlanta.

When the New York sender is attached, a signal is sent to the Atlanta sender to advise it that pulsing may proceed. It immediately sends the area code 207 to New York by MF pulsing and follows it with the remaining digits of the called number, AC4-2345, as they are received from the operator, ending with a start pulse, and then leaves the connection. All common control equipment in Atlanta is now free.

In New York, as soon as the Maine area code is received, it is submitted to the decoder. Upon examination of the code, the decoder finds that it is insufficient for routing purposes. New York has a direct circuit group to Portland over which traffic to some offices in Maine is routed, but other offices are reached through Bangor by way of Boston. In order to determine which route to take the decoder must know what office is desired. Therefore, it gives the sender a signal saying "come again when you have six digits" and leaves the connection. When the sixth digit arrives the sender again calls for a decoder and gives it the complete code 207-AC4.

The decoder again translates the area code, which now directs it to the foreign area translator which serves the Maine area, and submits the complete code to that translator. From the ensuing translation it learns that the route is by way of Boston and that all digits should be sent forward by MF. It then calls for a marker and releases the foreign area translator.

Subsequent operation is the same as previously described for Atlanta and the complete 10-digit number now arrives at Boston. At that point both codes are again translated since Boston also has a choice of routes to Maine, and the route to Bangor is selected. The translating equipment in Boston knows that Bangor is in the Maine area and that the area code will, therefore, not be needed. However, since Bangor is a TO having no senders, the Boston sender must pulse forward all of the digits needed to complete the call through switches in Bangor, Houlton, and Monticello. It is assumed that Houlton is arranged to route the call to Monticello on receipt of the digits AC4. Numerical digits 2345 will route the call through the Monticello switches to the called customer's line. These digits are all registered in the Boston sender but the digits required to switch the call through Bangor are not and must be supplied. An arbitrary set of digits beginning with 1 can be used for this purpose since no office code begins with 1 and there will, therefore, be no conflict.

The decoder in Boston, therefore, gives the sender the proper set of arbitrary digits, say 16, to be placed ahead of the office code AC4. The sender sends forward by the DP method 16-AC4-2345 driving switches in Bangor, Houlton, and Monticello to the called subscriber's line, and ringing starts automatically. The talking connection is now established and the common control equipment at all intermediate points is free.

When the called subscriber answers, the Atlanta operator's cord lamp is extinguished. When he hangs up, the lamp lights to denote end of conversation. The removal of the operator's cord automat-

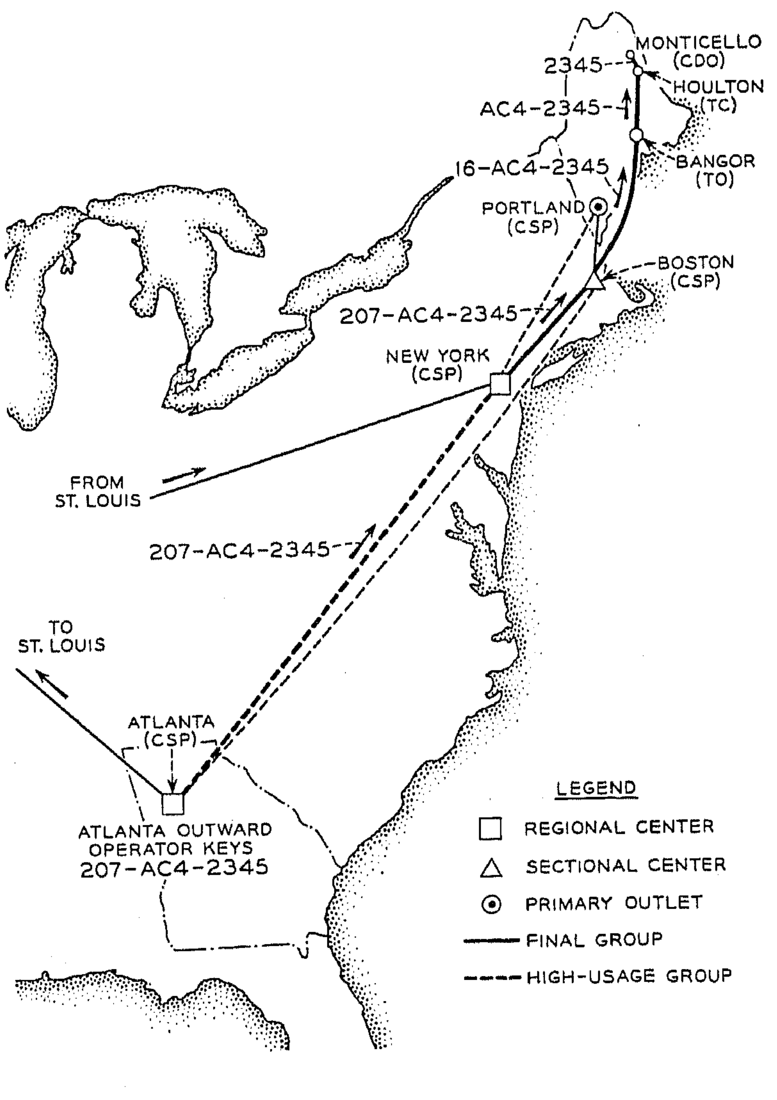
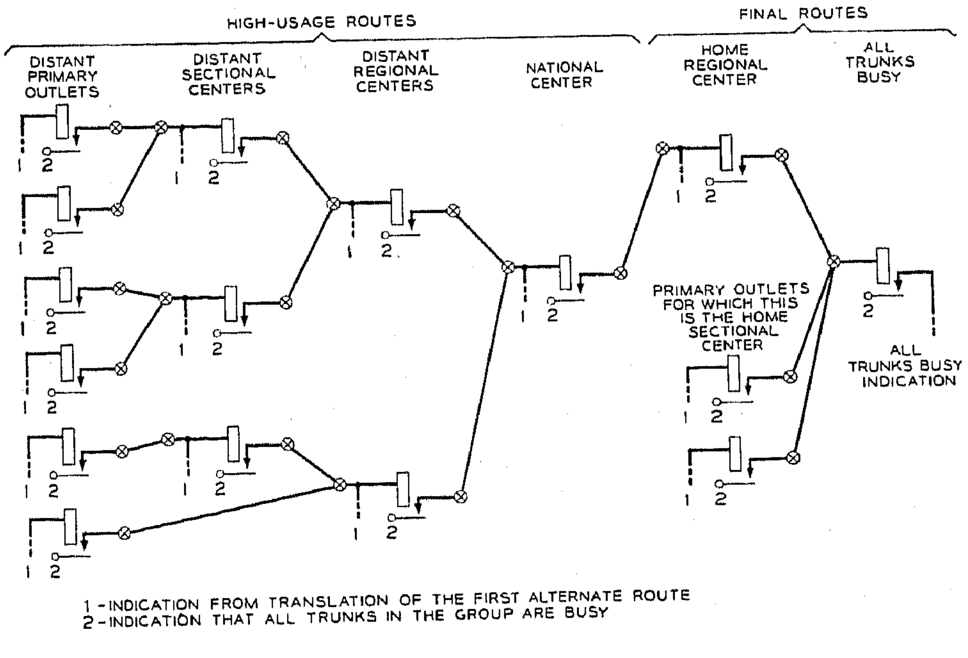


Figure 2 (left). Call from Atlanta, Ga. to Monticello, Maine

Figure 3 (below). Alternate route array for the decoder at a sectional center


HIGH-USAGE ROUTES
FINAL ROUTES
DISTANT PRIMARY OUTLETS
DISTANT SECTIONAL CENTERS
DISTANT REGIONAL CENTERS
NATIONAL CENTER
HOME REGIONAL CENTER
ALL TRUNKS BUSY
1
2
PRIMARY OUTLETS FOR WHICH THIS IS THE HOME SECTIONAL CENTER
ALL TRUNKS BUSY INDICATION
1 - INDICATION FROM TRANSLATION OF THE FIRST ALTERNATE ROUTE
2 - INDICATION THAT ALL TRUNKS IN THE GROUP ARE BUSY

ically releases the entire connection, the release of each link causing the next in line to release.

In setting up this call all of the characteristic CSP features were employed, automatic alternate routing in Atlanta, 6-digit translation in New York and Boston, digit storing, and variable spilling at all CSP's with substitution of arbitrary digits for the area code at Boston.

Transmission

All talking connections through the CSP system are made on a 4-wire basis, that is, separate pairs of conductors are provided for transmission in the two directions. This is done in order to simplify the problem of maintaining satisfactory balance so that the loss introduced by extra links in a connection can be held to a minimum value. The importance of this feature is emphasized by the fact that the switching plan permits as many as eight intertoll trunks to be connected in tandem for the completion of a call.

The advantages of 4-wire switching were fully explained in a paper[2] on the toll crossbar system now in service.

Signaling

In following the progress of the call from Atlanta to Monticello, Maine, it was observed that besides the transmission of information in the form of digits, it was necessary to pass a number of control and supervisory signals over the toll lines. These included seizure and disconnect signals in the forward direction and switchhook supervisory signals and sender attached signals in the reverse direction. On some calls it is necessary also to send flashing signals to indicate busy lines or trunks and ringing signals in either direction when operators are called in at intermediate or terminating points to assist in establishing connections.

For the early toll dialing installations, the signaling method most widely used was the composite method whereby signaling channels for the three circuits of a phantom group are derived from three of the conductors with the fourth being used for earth potential compensation. Direct current is used for signaling. This is a simple, reliable, and economical method of signaling and will continue to be used on circuits where it can be applied.

Where circuits are obtained from carrier systems, however, conductors are not available in sufficient numbers for signaling channels and other methods must be employed. Since carrier is used almost exclusively on the long-haul circuits, it was necessary to provide a signaling system to accompany it before toll dialing could be expanded beyond networks of limited range. To meet this situation a system[6] using a frequency of 1,600 cycles was developed and has been in service since 1948. Signaling is done by application and removal of the 1,600-cycle signaling current. The system is used in the same manner as the composite signaling system, to carry dial pulses as well as supervisory signals when used on circuits that require it. The set of leads brought out of the signaling unit are identical in function to those brought out of the composite signaling unit so that toll-line relay circuits will operate in the same manner with either type of signaling.

Since 1,600 cycles is in the voice range, the signaling current can be carried over the same channel that carries the speech current; but the signaling circuits must, of course, be protected against false operation due to speech, and precautions must likewise be taken to insure that the signaling tone does not interfere with speech. Protection against interference between signaling and speech is more difficult at 1,600 cycles than at higher frequencies because there is more energy in voice currents at the lower range. That value was chosen, nevertheless, so that it would be possible to operate over the narrow-band circuits that were established to relieve shortages occasioned by the war.

A new 2,600-cycle system to be used only on the broader band circuits has since been developed. It is simpler and more economical than the 1,600-cycle system. The older carrier systems, having been designed when practically all toll operation was by the ringdown method, made no provision for signaling since that was all done by short applications of the 1,000 cycles when there was no speech on the line. Some of the new carrier systems for short-haul applications are designed to provide their own signaling channel for each voice channel.

Principal Elements of the CSP System

CROSSBAR SWITCH FRAMES

Crossbar switches are used for incoming and outgoing link frames on which the trunks (both toll lines and trunks to and from local offices and switchboards) are terminated, and for sender link frames used to give trunks access to senders. These frames are similar to those in the toll crossbar systems now in service. Since they have been described in another paper,[1] they will be passed over with only a mention of their capacity.

Each incoming or outgoing link frame normally has terminals for 300 trunks. As many frames are provided as required for the size of the office. In the smaller offices, one train of switches with complete interconnection of incoming and outgoing frames is provided. In the larger offices, two trains each with its own set of markers are provided. When this is done, the incoming trunks are multipled to both trains and an extra build out bay is provided on the incoming frame to provide 400 terminals per frame. Since each train has a theoretical limit of 40 incoming and 40 outgoing frames, the maximum size of an office is theoretically 80 of each. Practical considerations, however, such as the number of markers that can be operated efficiently in a group and the maximum size office it is feasible to operate as a single administrative unit will limit an installation to about 60 incoming and 60 outgoing frames.

The sender link frame gives 100 trunks access to 40 senders.

SENDERS

Two separate groups of incoming senders are provided, one to receive DP and the other MF pulsing. Whether the system is installed in a step-by-step or a panel-crossbar area, both groups of senders always will be needed. MF will be received from senders in other CSP's and from switchboard positions. DP will be received from switchboard positions at TC's not equipped to send MF and in some cases from dialing A boards in the local area of the CSP itself.

Aside from the type of pulses received the functions of the two senders are identical. They have a capacity for receiving and sending eleven digits. They must register arbitrary digits given them by the decoder and send them out as directed. They will send out digits by either the DP or the MF method as required to control switches in distant offices, and in some installations also will send digits to an outgoing sender in the same office by the d-c key pulsing method, which employs direct current in various combinations of value and polarity through a pair of conductors.

When the CSP is in a panel-crossbar area a group of outgoing senders is provided to transmit either the type of pulses required by the equipment in local panel offices or the type used to reach manual offices.

MARKERS

The marker has been stripped of its usual translating functions and performs most of its duties on instructions from the decoder. It is told what leads to test for

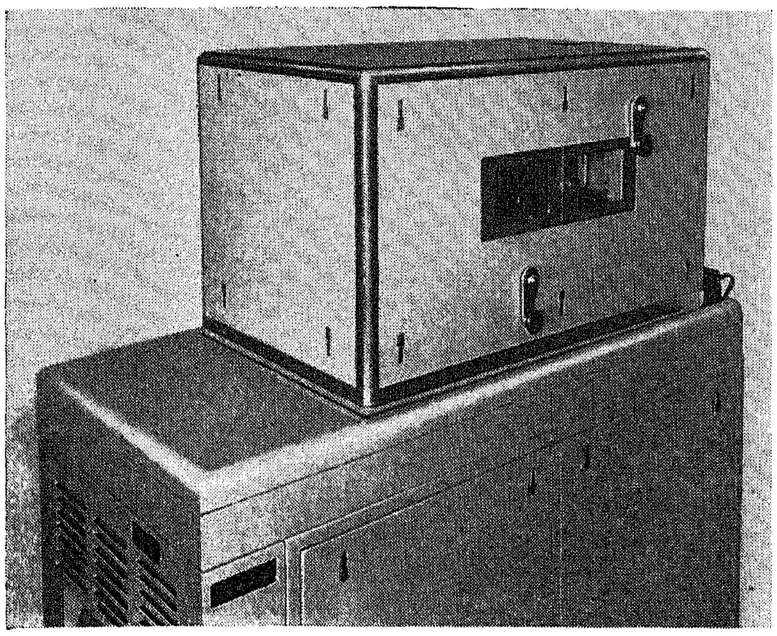


Figure 4. Card translator

idle circuits and where they are to be found in the trunk block connector, but having found an idle circuit it carries on the process of setting up the connection independently of the decoder. Having contact with both the incoming and outgoing trunks through connecting circuits, it determines what frames they are located on, connects itself to those frames, selects a path through them and sets up the connection.

In a single-train office, one group of markers common to the office is provided. In a 2-train office there is a group of markers associated with each train of switches.

DECODERS

A single group of decoders serves the entire office whether one or two trains of switches are provided. An important element of the decoder is the translator which will be discussed separately.

The decoder contains several hundred relays. A large group is used for registering the information furnished by the translator. Others use this information to control the action of the markers and senders.

One group of decoder relays which is of particular interest is the array used for automatic selection of alternate routes. It is composed chiefly of one relay for each CSP to which the office has a direct group of toll lines. The relays are arranged in an orderly pattern simulating the pattern of the CSP network for the country as seen from the CSP concerned and are interconnected in a pattern of progression corresponding to the fixed order of alternate route selection. Group busy leads from the toll line groups are connected to the contacts of the relays in such a manner that if a group is busy the relay corresponding to the next choice route in the chain will be operated. In this way the lowest choice route having an idle circuit will be selected speedily without testing individual trunks of separate groups. The decoder learns from the translator which relay in the array to operate first and the choice of the best route available follows automatically. The principle will be understood readily by reference to the simplified sketch in Figure 3. Contacts not shown on the relays cause the translator to select the route corresponding to the last relay operated in the chain.

TRANSLATORS

The magnitude of the translating job for nationwide dialing led to the decision to develop a new translator operating on a principle radically different from that employed in other crossbar systems. In previous systems, translation is done by relays. The code digits—never more than three—operate a group of relays which cause a single terminal corresponding to the code to be selected. A cross-connection is made between the code point and a route relay associated with the trunk group to be selected. The route relay has a number of contacts which are cross-connected to supply the information required for proper routing of the call. When changes in routing or equipment location of trunks within the office are made, it is necessary to change cross-connections.

With the nationwide dialing plan in operation, routing changes or opening of new offices in one part of the country will necessitate translator changes in many offices, some of them far removed from the scene of the event that forces them to be made. The changes in any one CSP will, therefore, be frequent and to make them by running cross-connections would be cumbersome and expensive. The new translator uses punched cards instead of relays, making it possible to effect changes by the simple process of removing old cards and inserting new ones in a machine. This can be done in a very short time and not only saves labor but requires less out-of-service time for the equipment. Figure 4 shows the machine.

A metal card about 5 by $10^{3}/_{4}$ inches is provided for each area code and also one for each office code that must be translated in a particular CSP, the cards representing destinations. The capacity of a single machine is about 1,000 cards. The cards are lined up in a box as in a filing drawer, with tabs along the bottom of the card resting on select bars which run the length of the box. One hundred and eighteen holes are punched out in all cards in fixed positions so that in the normal condition 118 tunnels are formed from one end to the other. A light source at one end of the box shines through the tunnels upon phototransistors (Figure 5) at the other end but the phototransistors are disabled until, concurrently with the dropping of a card, voltage is applied to them.

All tabs along the bottom of the card are cut off except those which serve to identify the particular card. When a code is presented to the machine a combination of select bars corresponding to the code is lowered. The card having all tabs cut off except those resting on the lowered bars will drop, but all other cards will remain in place. If nothing further were done, the dropping of the card would cut off all light channels; but on each card

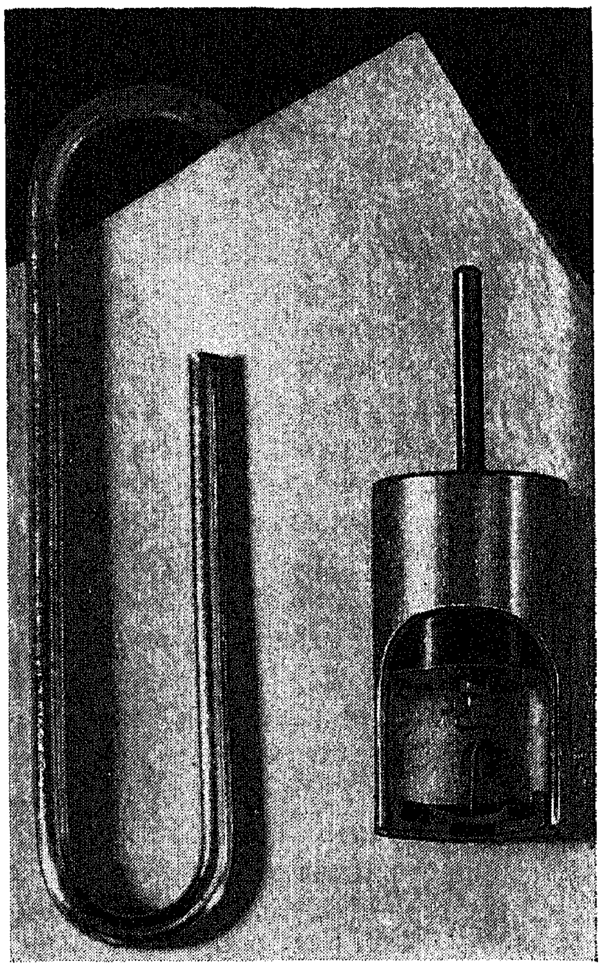


Figure 5. Transistor

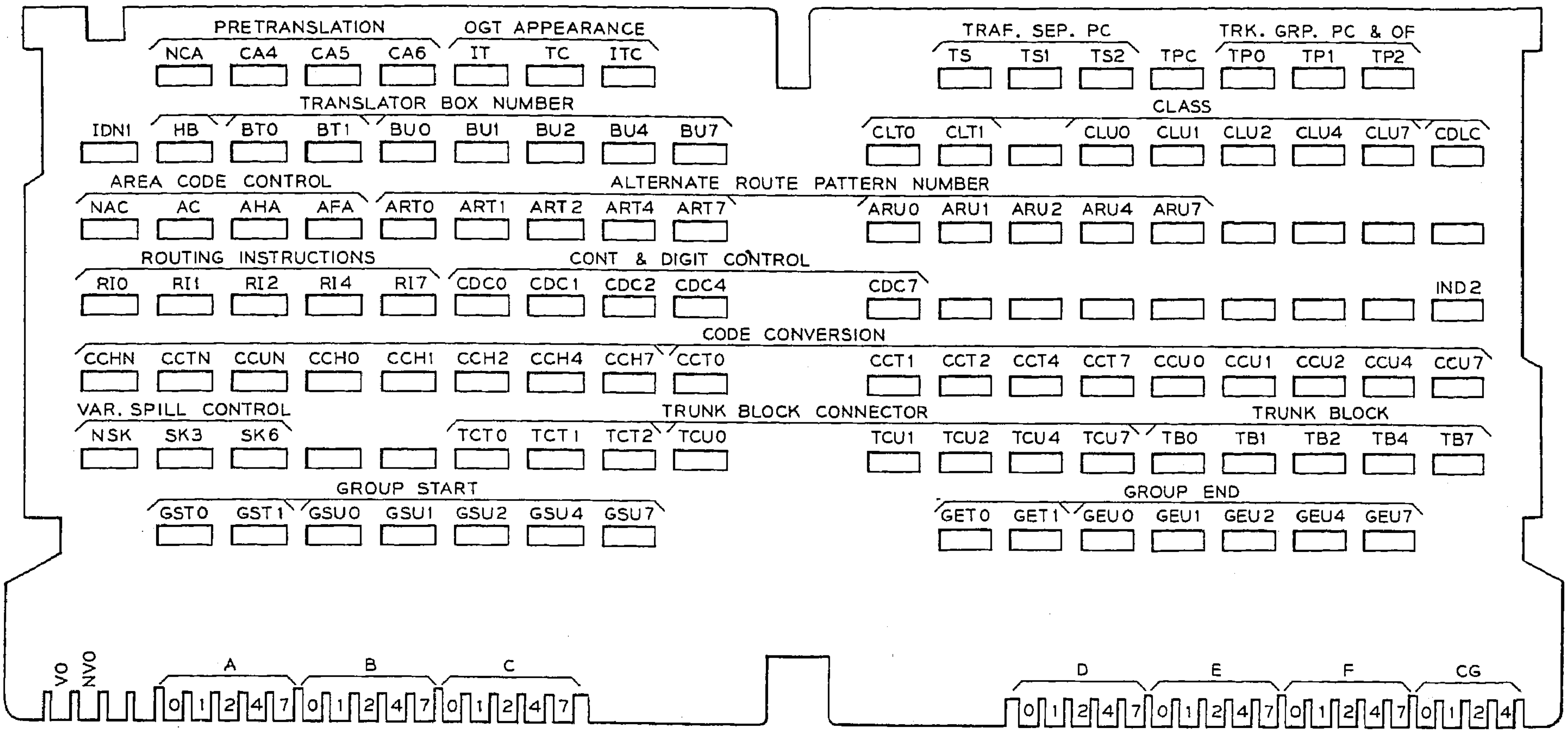


Figure 6. Card layout

some holes are enlarged and through these holes the light continues to shine, energizing the corresponding phototransistors. The combination of enlarged holes furnishes all of the information needed for routing the call to the destination represented by the card.

Figure 6 shows the functions of the various groups of tabs and holes. The designations will not appear on the actual cards. Figure 7 shows an actual card prepared for use.

Selecting Tabs—Input Information

The sole use of the information presented to the card translator is to enable it to select the proper card. The information presented is in the form of code digits with accompanying indications of their nature. The information is recognized by cutting off tabs along the bottom of the card in proper combinations.

The groups of tabs labeled A, B, C, D, E, and F are for the 6 code digits. For each digit used two tabs are left since the digits are registered in the sender on a two-out-of-five basis and the leads from the sender will operate the select bars directly. If the card represents an ordinary 3-digit code all tabs will be cut off except two each of the A, B, and C tabs, two of the four CG tabs and perhaps either the VO or NVO tab. The CG (card group) tabs are used in combination to indicate 3-digit, 6-digit, and alternate route card groups. The VO and NVO (via only and not via only) tabs are used when the group of toll lines over which the call will be routed is divided into one subgroup of a transmission grade suitable for only terminal traffic and another subgroup for either terminal or switched traffic. If the card represents an ordinary 6-digit code two tabs will be left in each of the digit positions, and a different pair in the CG group.

Punch Holes—Output Information

The output information from the card translator is recognized in the decoder and marker by relays operated in the combinations set up by enlargement of associated holes in the card. The output from the phototransistors is amplified by other transistors to fire cold cathode tubes which in turn operate the relays.

The pretranslation group on the top line of Figure 7 indicates how many digits the sender must supply for a complete translation. The term "pretranslation" implies that further translation is required. This is not always true. In many cases only the first three digits need to be translated and all information needed for routing the call is supplied by this card. In many cases, the six digits of the area and office code are needed and the routing information will be on another card to be selected later. For certain calls such as calls to operators, only four or five digits are needed. These are treated as 6-digit calls by having the sender supply the extra one or two digits to fill the complement. The NCA hole enlarged means "no come again," that is, three digits are sufficient and translation will proceed. The other holes enlarged mean respectively "come again when you have four, five, or six digits," and no further translation is done until the sender comes back a second time, probably to a different decoder, with an indication that it has the required number of digits.

The OGT appearance holes are used in a 2-train office to tell which train the outgoing trunk appears on and enable the decoder to select a marker in the proper group.

The remaining holes on the top line are for controlling operation of traffic meters.

The translator box number holes in the second line are punched on the area code cards to indicate which machine con-

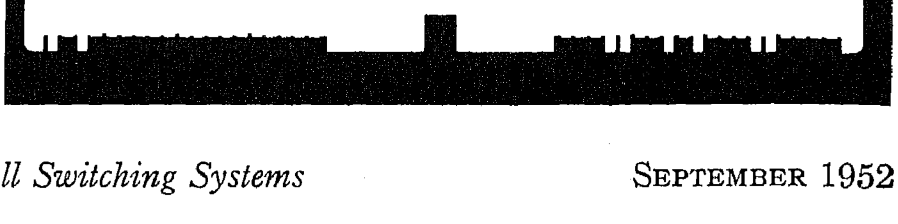


Figure 7. Punched card

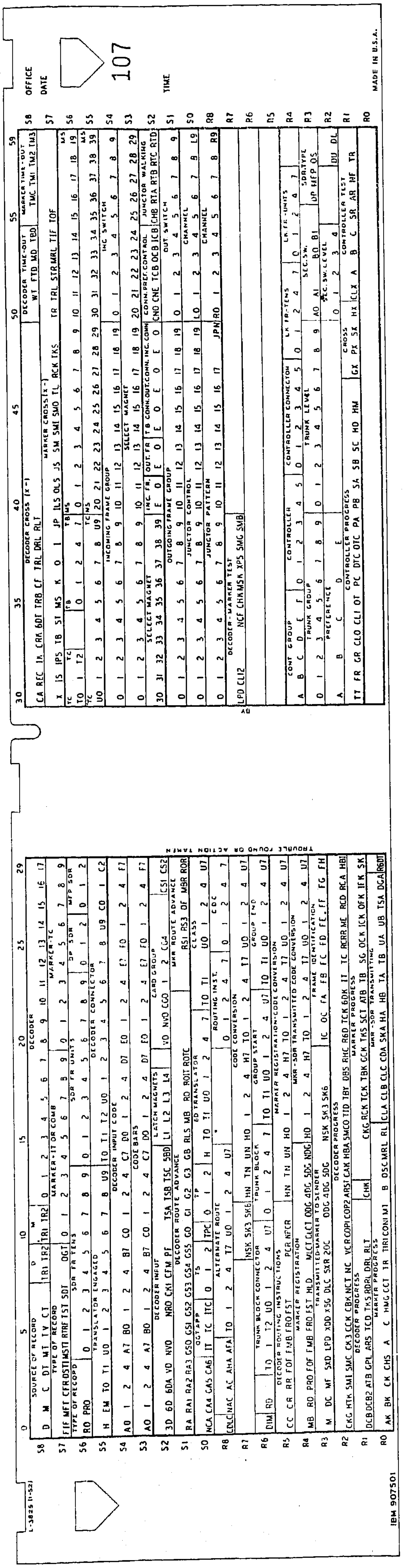


Figure 8. Trouble recorder card

tains the individual cards for the called area when 6-digit translation is required.

The IND1 hole in the second line and the IND2 hole in the fourth line are index holes and are never enlarged. Any card that drops will always cut off the light through those channels. This serves as an indication that a card has actually dropped and that the phototransistors associated with the other holes should be prepared for action. The index holes also aid in trouble detection and in proper disposition of calls where cards are deliberately omitted or where operators have dialed a blank code in error.

The class holes indicate such things as type of pulsing and nature of the signaling channel used on the trunk group out of the office.

The area code control holes in the third line are to tell the decoder what to do about dropping or spilling forward an area code registered in the sender or supplying an area code when none is registered. This information is needed primarily in connection with alternate routing. The alternate route pattern number holes tell the decoder at what point to enter its chain of alternate route relays for the first choice alternate. Provision is made for a maximum of 100 entry points.

The holes on the fourth line are for making proper dispositions of calls when no circuits are available on any routes, telling how many digits to expect on certain calls and other items of a detailed technical nature.

The code conversion holes on the fifth line supply the arbitrary digits to replace code digits on calls routed through step-by-step TO's or TC's. Provision is made for one, two, or three digits as required.

The variable spill control holes in the sixth line tell whether to spill all digits received, skip the first three code digits, or skip six code digits.

The remaining holes define the location on the equipment of the test leads for the trunk group over which the call will be routed.

The notches around the edges are used for positioning and removal of cards.

An individual card is removed from the stack by first keying the code to cause it to drop so that it may be identified. Since a card can be located easily in this manner, it is unnecessary to keep cards in any ordered position in the box.

At least one translator is provided in every decoder. It contains the cards for all offices in the home numbering area of the CSP, for certain operator codes, the single 3-digit card for each toll numbering area and a card for each toll line group out of the office that can be used as an alternate route. If there are other areas to which the volume of traffic is very high and for which 6-digit translation is required the cards for those areas are put in a second machine in each decoder. Cards for other areas are put in foreign area translators common to the office and accessible to all decoders on a one-at-a-time basis. An emergency translator is provided to permit removal of all cards to it from any translator which may require prolonged repair work.

TRAFFIC CONTROL PANEL

The traffic control panel is located in the operating room. The equipment in it consists of a key for each group used as an alternate route. When a particular key is operated, no alternate routed traffic will be offered to the group represented by it nor to any group above it in the fixed alternate route pattern. This is done to relieve offices which are overloaded by either unforeseen or predicted traffic peaks.

Maintenance

The maintenance facilities for the new CSP system are basically similar to those of the older toll crossbar system with the necessary addition of equipment to test the new features introduced. The sender test frame is, of course, obliged to test the CSP features added to the sender and the trouble indicator frame is changed to operate with the new decoders, translators, and markers.

In place of the lamp trouble indicator the new trouble recorder introduced with the latest local crossbar system[7] is used. Whenever trouble is encountered, it punches on a card a record of the circuits involved and of the important events that had occurred in the progress of the call, as an aid to the maintenance man in locating the trouble. A sample trouble recorder card is shown in Figure 8.

Automatic equipment for testing the operation and transmission features of intertoll trunk has been designed for the older systems and for the new CSP system.

Switching Aspects of Customer Toll Dialing

In the course of developing the switching system for CSP's, the requirement for handling long-distance traffic dialed by customers as well as that dialed by operators was kept in mind. The trial of long-distance customer dialing now in progress in Englewood, N. J. confirms the soundness of the basic plan and exemplifies the principles involved in full realization of the plan. With a toll network laid out to accept a distinctive 10-digit number for any telephone in the country and route it to the proper destination, the remaining tasks to be performed are to provide for delivery of the number to the toll network from the customer's dial instead of from an operator and to provide an automatic record of the call for charging purposes.

In Englewood both tasks were performed quite easily. The Englewood local office equipment is of the most modern type[7] and includes automatic message accounting (AMA)[8] facilities. When it was in the development stage, the ultimate requirement for nationwide customer dialing was foreseen and provision was made for expanding the digit capacity of the switching equipment at small expense. Also the designs of the accounting center were such that corresponding changes could be made readily. In the new local office switching system, arrangements were included for sending forward the complete number, as received, to the toll office by MF pulsing. The system also was designed to be capable of automatic alternate routing and this feature is used in the trial.

Expansion of the program will, of course, demand that similar arrangements be provided for the older types of local switching systems already in service More extensive modification will be required to make them capable of giving the customer the same service. For them, as for the most modern system, however, AMA equipment is admirable for recording the information necessary for charging for the calls.

The requirement for customer toll dialing that senders (or directors) and recording equipment be provided has a bearing on the type of equipment used at TC's and TO's. For calls handled by operators and for calls received by the customers through such offices the only disadvantage of step-by-step equipment without senders at those points is that the CSP equipment at other points must be somewhat more complicated and expensive than it would otherwise need to be. But with customer dialing, if senders and recording equipment are not provided either in the local office or in the TC or TO, the calls must be routed by the most direct means possible to a CSP where such equipment is provided. Thus some advantages that might be gained from having them at the TC or TO would be lost:

1. In some cases an indirect route to the CSP would need to be taken for the sole purpose of recording the call. For example, a call which normally might be switched from a TC through a TO to another TC would need to be connected to the CSP for making the record.
2. There is no operator at the TC or TO to select an alternate route and with the equipment there incapable of automatic alternate routing, the economies and service protection inherent in the alternate routing procedure would be lost.

If step-by-step toll switching equipment is already provided at a TC or TO, senders (or directors) could be added, making it in effect a common control switching system. This measure would permit automatic alternate routing and the further addition of recording equipment would eliminate the indirect routings for recording purposes.

A further benefit from having common control equipment in TC's or TO's can be realized in some instances. When a customer is served by a local office that has no senders, he must dial one or more directing digits (probably three digits) ahead of the 7- or 10-digit number in order to get to an office where senders are provided. It is, of course, desirable to avoid this extra burden on the customer. Where the equipment in a TC or TO can be used in common for switching local and toll traffic, the customers whose lines are terminated in that office will be dialing directly into senders, if the equipment uses common control, and will, therefore, benefit in that they will not have to dial directing digits.

Conclusion

The new system was designed to implement the nation-wide switching plan, which integrates the switching network of the entire nation into a single unit. This switching job, requiring a high order of mechanical intelligence, is the most comprehensive ever performed by any system. Skillful manipulation of code digits enables the provision of a numbering plan covering the entire country with a minimum number of digits to give each customer a distinctive number. It also obviates the need for extra expense to make step-by-step toll offices satisfactory operating elements of the plan in locations where CSP features are not essential.

The automatic and almost instantaneous selection of alternate routes makes it possible to give virtual no-delay service without greatly increasing the cost of outside plant and to make multiswitch connections at a speed comparable to that for local service.

The translating equipment simplifies administration of the plan which demands co-ordination of activities on a nation-wide basis.

The numbering plan, the switching plan, and the CSP equipment which implements them make it feasible to offer nation-wide dialing service to customers without the aid of operators when automatic charging facilities and local office switching arrangements for handling the three extra digits of the national number are provided. It will be readily appreciated that so far as the CSP switching equipment is concerned, it is immaterial whether the digits it receives come from an operator or from a customer. The call will be routed to its destination and supervision for charging purposes will be furnished in the same manner in either event.

The new system represents an important step in the process of continually improving the long-distance switching methods of the Bell System with consequent improvement of the service to all telephone customers in the United States and Canada.

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No Discussion

Dynamic Hysteresis Loop Measuring Equipment

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WITHIN the past 2 or 3 years many scientists and engineers working on research and development projects in the magnetic amplifier field have come to realize that the dynamic hysteresis loop of a magnetic material plays an important role in determining the amplification and control characteristics of high-gain magnetic amplifiers. Figure 1 shows a block diagram of an excitation and test circuit which often has been used to display the dynamic hysteresis loop of a core material upon the screen of a cathode-ray oscilloscope tube.

This circuit includes means for impressing upon the vertical, or B, axis of the oscilloscope a signal which is proportional to the flux density in the core and impressing upon the horizontal, or H, axis of the oscilloscope a signal which is proportional to the magnetizing force acting on the core specimen under test. The B axis channel includes a secondary (B) coil wound on the specimen under test, an integrating circuit, and a linear amplifier, the output of which supplies voltage to the vertical deflecting plates. The H axis channel consists of an amplifier to amplify the relatively low voltage drop across a resistor (current shunt) in series with the primary or exciting (H) coil so as to provide an adequate deflection voltage to the horizontal deflecting plates of the oscilloscope. The Lissajous figure shown by this system on the cathode-ray tube screen then will be indicative of the shape of the dynamic hysteresis loop of the specimen under test.

Figure 2 shows a 60-cycle dynamic hysteresis loop of Deltamax as displayed in the manner just described, by a good system of amplifiers and integrator circuits. Furthermore, by suitable calibrating means, the amplitude of the deflection in the vertical axis will indicate the average flux density in the core and the amplitude of the horizontal deflection will indicate the magnetomotive force applied to the core material.

On many occasions the hysteresis loop displayed by such a system has been found to show some very peculiar shapes. At such times one then begins to question the fidelity of the system being used to display the dynamic hysteresis loop.

This paper will endeavor to indicate the several sources of errors and their effects upon the displayed hysteresis loop. Also, it will indicate means for compensating for certain errors introduced by the amplifiers and will specify the general performance characteristics of amplifiers which will be satisfactory for displaying a dynamic hysteresis loop without introducing serious errors and/or distortions.

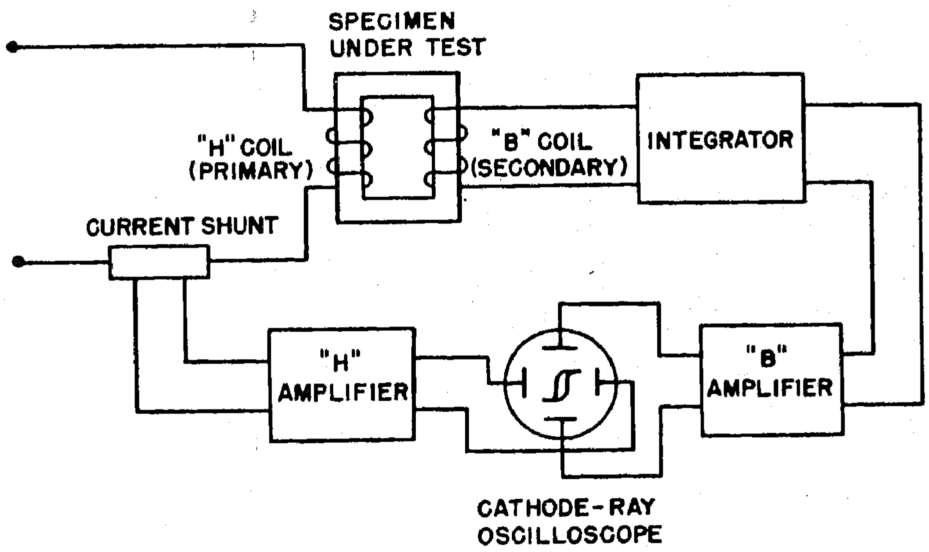

Figure 1. Block diagram of excitation and test circuit

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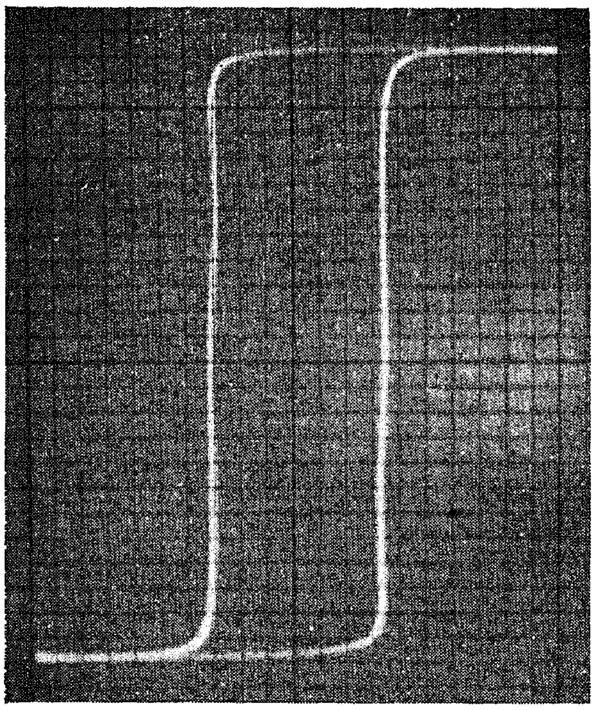

Figure 2. Dynamic hysteresis loop of 0.005-inch Deltamax at 60 cycles

Some Sources of Errors, Their Effects upon the Displayed Hysteresis Loop, and Means for Minimizing Them

To provide a readily reproducible method for testing cores in a manner approximating their use in magnetic amplifiers, a sinusoidal flux wave has been chosen. If the wave form of the average flux density in the core departs materially from that of a sine wave, the maximum rate of change of flux in the core will exceed that of a sine wave. Higher rates of change of flux will produce higher values of indicated dynamic coercive force and, therefore, will be in error. If the total harmonic distortion of the voltage induced by the flux wave is kept to less than 10 per cent, errors due to this effect will be low enough to be tolerable. Figure 3 shows an induced voltage wave with 6-per-cent total harmonic distortion due to saturation effects.

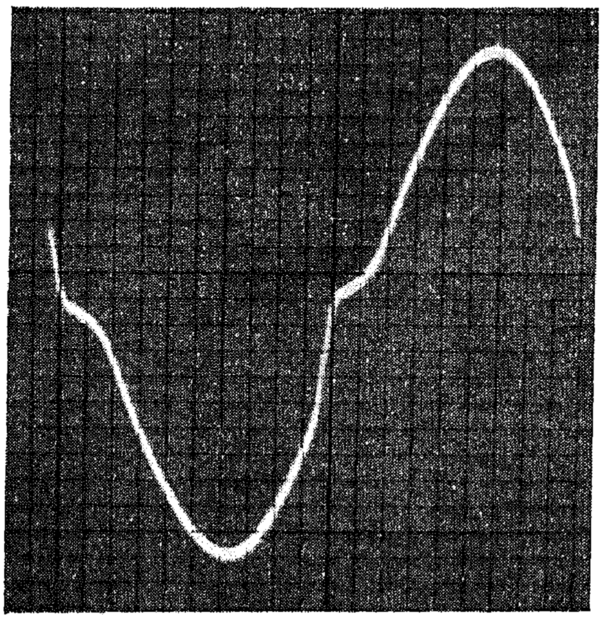

Figure 3. Voltage wave with 6-per-cent harmonic distortion

The source of power used to excite the specimen under test should be a good sine wave with low distortion. It should have a low internal impedance since it must supply highly distorted current wave shapes with only slight distortion of the voltage wave shape of the power source. The resistance of the exciting coil or primary should be kept low.

The current drawn by the exciting coils of the specimen often is highly distorted and may have very high ratios of peak to average values. The resistor used in series with the excitation circuit to act as a current shunt for supplying voltage to the *H* amplifier must be restricted to relatively low values of resistance. The peak voltage drop across this resistor, plus that across the resistance of the exciting coils, must be only a small fraction of the total voltage applied to the excitation circuit. Excessive nonsinusoidal voltage drops across these resistances will cause the voltage wave form impressed upon the excitation coil to be distorted seriously at peak flux densities which saturate the core.

The *B* coil, or secondary winding, should be fairly closely inductively coupled to the primary winding. However, care should be taken that the coupling capacitance between this winding and the excitation winding be kept low. This is particularly necessary when testing rectangular hysteresis loop core materials, since high-frequency components of voltage are present when testing under conditions of high saturation. Although these high-frequency components of voltage are small, excessive capacity coupling will show up as small oscillations superimposed upon the normal hysteresis loop.

The integrating circuit should have a high impedance in order that the effects of its loading upon the *B* coil be kept low. Excessive loading by this circuit appears as a widening of the hysteresis loop and therefore is another source of error in determining the true values for the dynamic coercive force and the true shape and area of the dynamic hysteresis loop.

The harmonic distortion contributed by the amplifiers should be held to less than 2 per cent. The gain versus signal level should be substantially constant over the operating range.

The *H* axis amplifier must be a reasonably broadband amplifier with low phase-shift response characteristics at the test frequency. Harmonic frequencies present in the exciting current may include values as high as the 20th harmonic when testing rectangular hysteresis loop materials. The phase shift of these harmonics must be kept low to prevent the portion of the hysteresis loop above the knee of the curve from appearing as shown in Figure 4. The phase shift of the amplifier at the testing frequency and below also should be kept low. Excessive low-frequency phase shift causes distortions as shown in Figure 5.

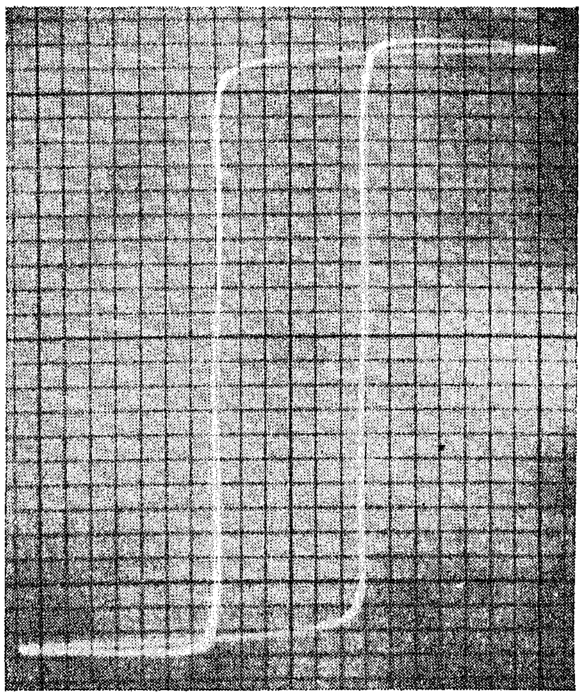

Figure 4. Distortion of loop due to excessive phase lag in H amplifier at high frequencies

Care should be taken to assure that this amplifier will not be overloaded by the high peak voltages present in the voltage drop across the *H* axis dropping resistor when testing to high saturations. The high ratio of peak-to-average of this voltage may overload this amplifier and cause errors in indication of the peak values of *H* under which the specimen is being tested. In some cases, clipping of this voltage is desirable to obtain accurate values of the dynamic coercive force under conditions of high peak values of

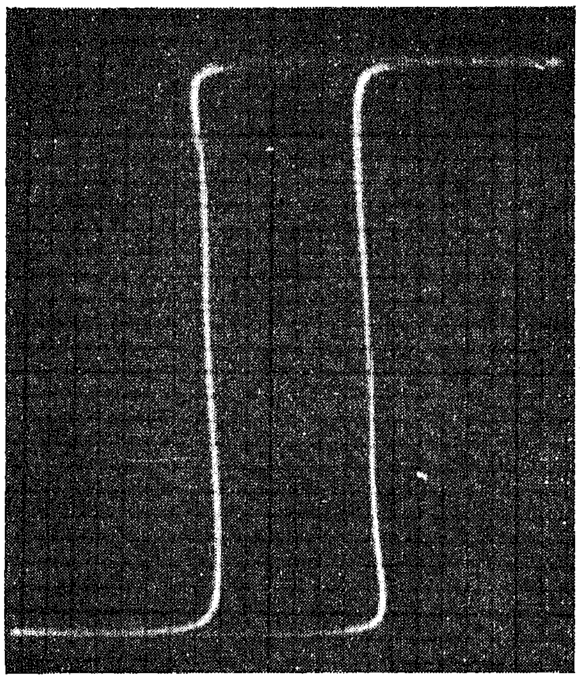

Figure 5. Distortion of loop due to excessive phase lead in H amplifier at low frequencies

magnetomotive force. However, any such clipping circuits should not effect the calibration or voltage gain of the amplifier within the desired range of linear response.

The voltage response characteristic of the B axis amplifier at frequencies above the test frequency is of secondary importance. However, the phase response characteristic at and below test frequency is very important. Excessive low-frequency phase shift (leading phase angle) causes a distortion of the loop of the type shown in Figure 6. Note that this is very similar in appearance to Figure 4 for the case in which the high-frequency phase shift of the H amplifier is excessive.

Figure 7 shows the effect of excessive high-frequency phase shift (lagging phase angle) of the combined B amplifier and integrator circuit. Note that the effect of this is to widen the loop tip instead of narrowing it or causing a reversed loop effect as shown by Figures 4 and 6.

If the frequency of the power source supplying the amplifier tube filaments and the plate supply rectifier circuit differs from that of the testing frequency, then hum effects are detected readily by a blurring or widening of the oscillograph trace over the entire figure being displayed by the oscilloscope. However, when the test frequency is synchronous with the source supplying power to the amplifier, the effects of hum in the amplifier are not so readily apparent. Hum effects from the amplifier filaments often cause symmetrical distortion of the hysteresis loop display. Its effect may be either a widening or a narrowing of some portion of the loop. The portion of the loop at which it appears depends upon the phase of the hum with respect to the excitation voltage. This type of hum effect usually can be detected by reversing the phase of the voltage applied to the excitation circuit and noting any change in the shape of the displayed figure.

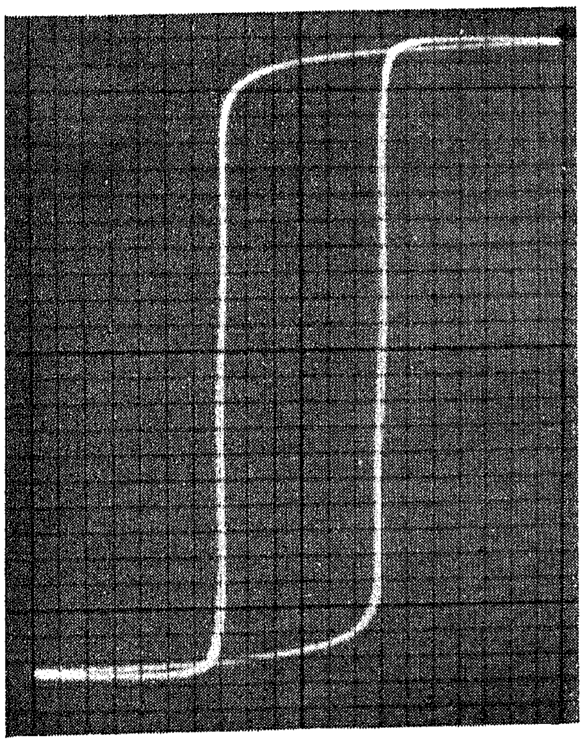
Figure 6. Distortion of loop due to excessive phase lead in B amplifier at low frequencies

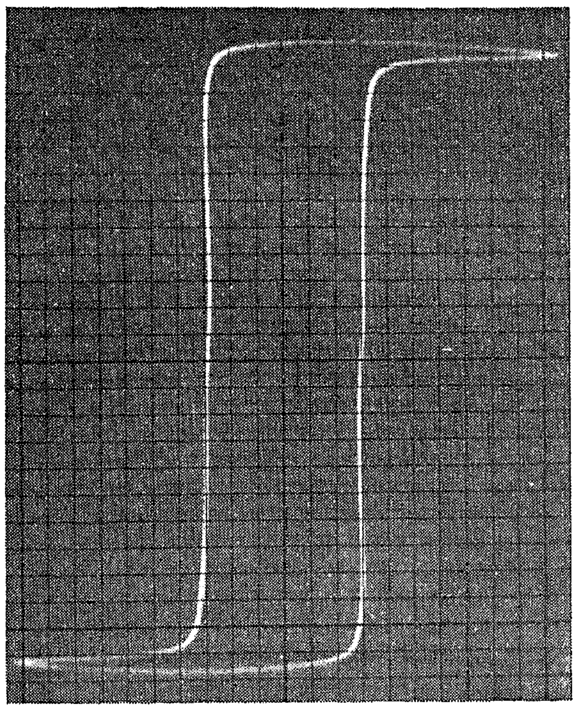
Figure 7. Distortion of loop due to excessive phase lag in B amplifier plus integrator at high frequencies

Hum due to insufficient filtering of the plate supply to the amplifier has a predominant second harmonic component. This causes an asymmetrical distortion of the hysteresis loop and is evidenced by a reduced deflection in one portion of the loop and increased deflection in the corresponding portion of the other half of the loop.

Criteria for Maximum Tolerable Errors

In order to specify the minimum performance requirements of measuring equipment such as this, it is necessary to set up some criteria upon which to judge whether or not the equipment is satisfactory. The criteria chosen for this dynamic hysteresis loop measuring equipment are as follows:

All amplitude or deflection distortion effects were judged as being satisfactorily low if the error they introduced was less than 5 per cent of the indicated value or less than one trace-width, whichever is greater.

The phase distortion was considered unsatisfactory if it was detectable by a close observation of the displayed figure shape. In many cases this amounted to a change corresponding to approximately the width of the oscillograph trace. A rectangular hysteresis loop material was used as the specimen under test when judging phase distortion effects upon the measuring circuit performance. This imposes severe requirements upon both the B and H channels. The H amplifier must handle a very wide frequency range and a phase distortion in the B amplifier is detected readily by a widening or a reverse looping of the trace above the knee of the hysteresis loop.

Characteristics of Amplifiers of Minimum Performance

The minimum performance characteristics required to meet the criteria specified will be indicated now. Each amplifier is assumed to be a 3-stage amplifier and is specified upon the basis that a good wide-band amplifier is used for the other axis. Somewhat poorer phase response characteristics could be tolerated if both amplifiers were identical and had a constant time delay. However, this possibility has been ignored in arriving at the requirements given here.

To keep the phase shift to a satisfactorily low value, each stage of the H axis amplifier should have less than 45 degrees of phase shift at a low frequency of 1/100th of test frequency and at a high frequency of 250 times test frequency. With conventional types of resistance-capacitance coupling networks and with screen and cathode circuits such that they do not contribute to the frequency response characteristics, the amplitude response per stage will be down 3 decibels at the frequencies just indicated.

The requirements for the B amplifier phase shift are very stringent on low-fre-

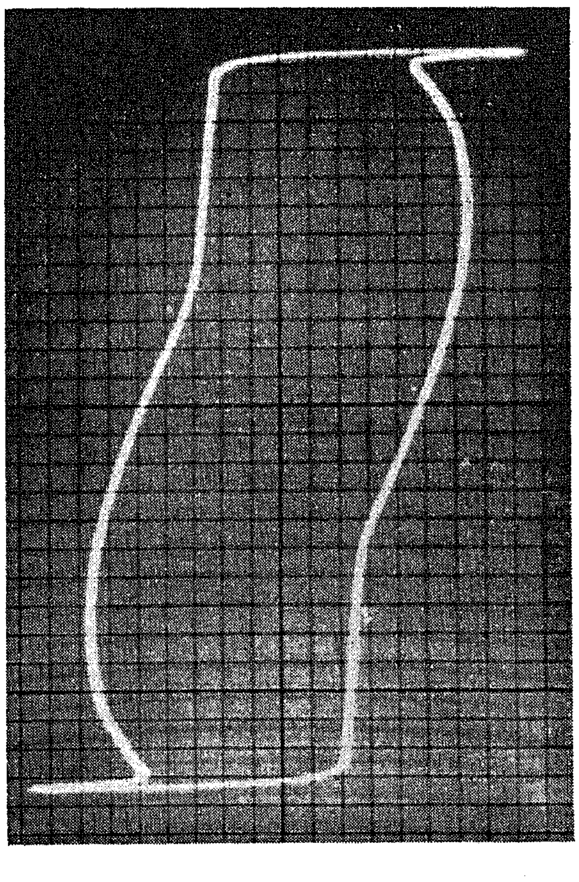
Figure 8. Dynamic hysteresis loop of 0.010-inch Deltamax at 400 cycles

quency response and a little less stringent than those of the H amplifier on the high-frequency end. If a single resistance-capacitance integrating circuit having a phase angle at operating frequency of 89.75 to 90 degrees is used, then the low-frequency phase response per amplifier stage must have a phase shift of less than 45 degrees at 1/300th of the operating frequency. However, if suitable compensation is provided by an additional corrective integrating network per amplifier stage, then the low-frequency phase response need be only less than 45 degrees at 1/30th of the operating frequency.

The high-frequency phase response of this B amplifier should be such as to have less than 45 degrees of phase shift in each stage at 100 times operating frequency.

With a suitable broad-band B amplifier, a simple single resistance-capacitance network of proper values will provide satisfactory integration. It should be such as to provide, across the capacitor, a voltage which lags the input voltage by 89.75 to 90 degrees at operating frequency.

When additional integrating types of resistance-capacitance networks are added to equalize for excessive low-frequency phase advance or lead due to the coupling networks in the amplifier, the equalizing networks should be such as to provide, at operating frequency, a total over-all lagging phase of 90 degrees ± 0.25 degree for the combined integrator and entire B axis amplifier circuit.

The maximum distortion in the voltage wave induced by the flux wave should not exceed 10 per cent. This includes distortion due to the power source and all series resistance in the excitation circuit.

As a final check for a complete system of these minimum performance specifications, networks representing this complete amplifier system were inserted between a test specimen and a broad-band oscilloscope. The loop displayed by this system was photographed and compared with photographs of the loop displayed when using a 90-degree integrator network and just the broad-band oscilloscope. The dynamic hysteresis loop displayed by the complete system met the original criteria set forth when, at test frequency, the integrator circuit was adjusted so that the phase of the output voltage of the B amplifier lagged the phase of the output voltage of the H amplifier by no less than 90 degrees nor more than 90.5 degrees.

The amplifier performance specified here is admittedly somewhat conservative. However, when one employs amplifiers meeting the specifications, along with carefully applied instrumentation, and obtains a dynamic hysteresis loop such as that shown by Figure 8, it is safe to assume that the displayed Lissajous figure truly represents the dynamic hysteresis loop of the specimen under the conditions imposed by the test circuit.

No Discussion

A New Type of Magnetic Servo Amplifier

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MEMBER AIEE

Synopsis: This paper describes a new 2-stage magnetic amplifier for high-performance servomechanisms, particularly to be used in connection with those types of 2-phase induction motors having such a low ratio of effective viscous friction to inertia that it is difficult to stabilize at high gain in a closed loop. The basic idea consists in feeding the amplifier field winding of the a-c reversible motor by a variable unidirectional current having a fundamental frequency (60 or 400 cycle) component and a d-c component producing an effective dynamic braking of the motor, preferably in such a way that the d-c component increases when the a-c component decreases, and vice versa.

There are various possibilities for designing of magnetic servo amplifier circuits which are based upon the aforementioned principle. However, only two typical 2-stage circuit arrangements will be treated in this paper to show the unique properties of these circuits which have inherent dynamic braking characteristics.

ANY closed-loop servomechanism is inherently oscillatory. In principle, the use of amplification in such systems is seldom possible without introducing a certain time-lag between cause and effect (due, for example, to the inductance of a field coil arrangement), which gives rise to hunting. The reason for the instability is that the correcting means persists a little too long, hence the error is overcorrected. A reversal of the correcting means follows, which again overshoots, and, unless some antihunt method is employed for cancelling the tendency to swing, it will continue indefinitely.

Unfortunately, the use of magnetic amplification is associated with a certain additional time-lag of the magnetic amplifier itself. However, it should be born in mind that the lags arising from external parts of the servomechanism are often appreciably greater than those inherent in the magnetic amplifier. For example, when working with a power supply frequency of 400 cycles per second, the usual response time of about 6 to 20 cycles (15 to 50 milliseconds) represents a performance which will not cause any difficulty in many cases. Of course, if the magnetic servo amplifier is designed to operate from a power supply of 60 cycles per second, a response time of 6 to 20 cycles (100 to 333 milliseconds) may not be acceptable. Thus, in this case, a very short response time of about 1 cycle only is really desirable.

F. G. Logan[1] has shown that this may be accomplished in various ways by using a-c controlled self-saturating circuits, the alternating control current having preferably the same frequency as that of the power supply generator. Other types of self-saturating circuits having a response time of 1 cycle only have been described by R. A. Ramey[2,3] and A. E. Schmid.[4] An inherent characteristic of such circuits is that control is achieved in full during each cycle and that when the controlling current is changed, full response to this change is reached not longer than 1 cycle later. Of course, application of these self-saturating circuits having inherent 1-cycle response will be particularly valuable for the design of magnetic servo amplifiers to be operated from a 60-cycle-per-second power supply.

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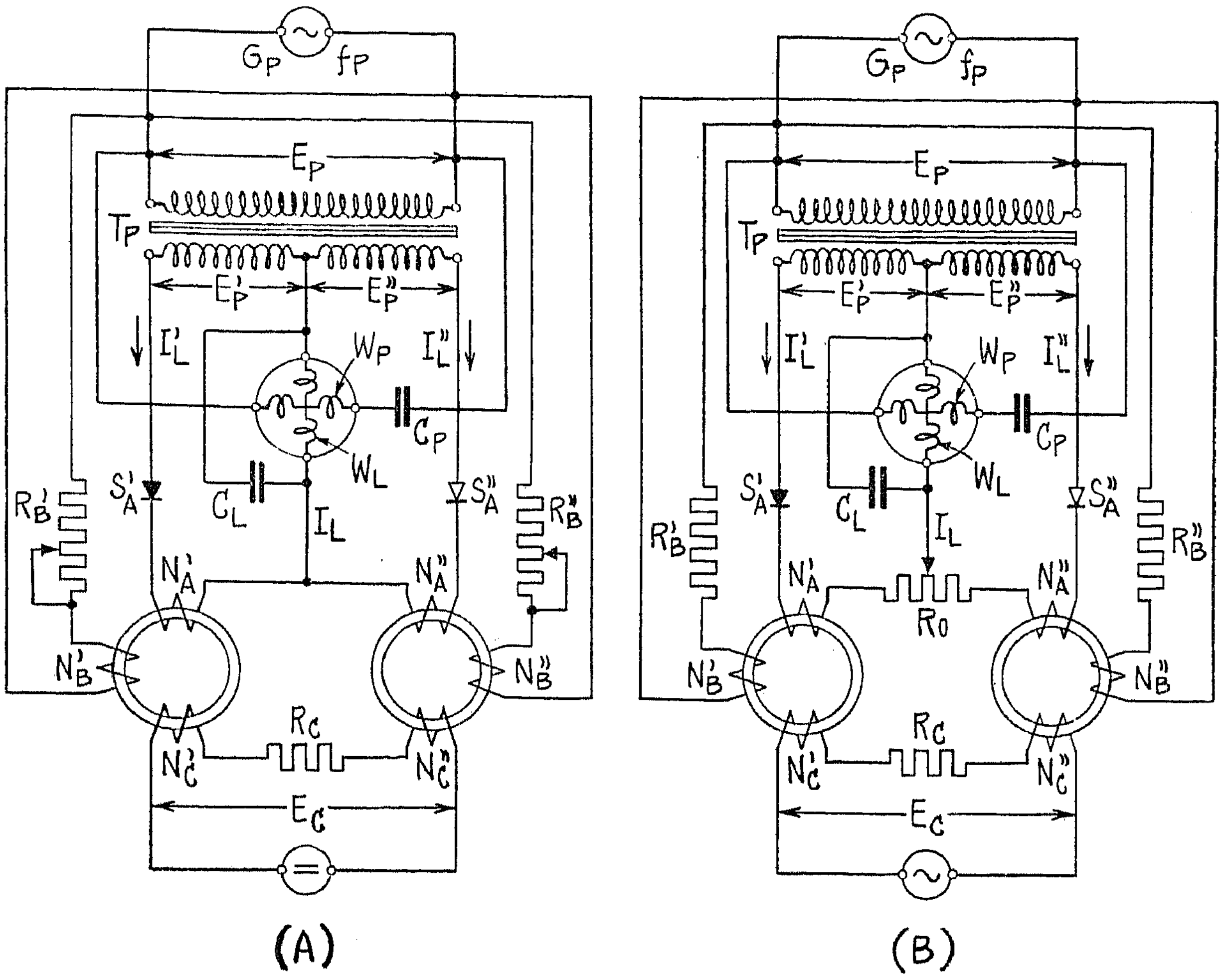


Figure 1. Two types of full-wave push-pull output-stage circuits having inherent dynamic braking characteristics with d-c control windings (A) or with a-c control windings (B)

Common remedies for hunting are damping the motor itself by introducing dynamic braking, application of a resistance-capacity differentiating network or tachometer to stabilize by means of derivative feedback, and magnetic or viscous flywheel dampers.[5] The purpose of such devices is to provide damping at a rate appropriate to the requirements of stability.

Another important factor governing the satisfactory operation of a closed-loop servomechanism is the proper adjustment of the sensitivity or gain of the magnetic amplifier. The combination of the error signal voltage per degree and the actual amplifier gain governs the tightness of the system. If the gain is too great, then the servomechanism will oscillate in spite of application of an effective antihunt method. It is often desirable to adjust the gain of the magnetic servo amplifier in such a way that not more than one overshoot, or no overshoot at all, will be obtained.

Referring to the damping properties of 2-phase induction motors which are used extensively in high-performance servomechanisms, it is important to stress that some of these motors have good viscous friction to inertia ratios or special damping means,[5] and others have not. Separately excited induction-meter type reversible motors, as sometimes used in self-balancing potentiometer recorders,[6] have a permanent magnet[6] producing eddy-current damping. Some types of low-inertia squirrel-cage induction motors and drag-cup type motors have eddy-current damping devices built into the motor. There are, however, other constructions of such motors which have virtually no damping properties so that additional damping methods must be applied to prevent hunting.

The use of vacuum-tube amplification offers the possibility of introducing effective dynamic braking of the motor by means of the d-c component flowing through the plate circuit, which contains the amplifier field winding of the 2-phase induction motor. However, if such a motor is to be controlled by means of a magnetic servo amplifier, considerable practical difficulty will be caused by the fact that the alternating load current of the conventional magnetic amplifiers of the balance detector type[6] does not have a d-c component which may be used for producing dynamic braking of the motor.

This paper describes a new type of 2-stage magnetic servo amplifier, for use particularly with 2-phase induction motors having very poor damping properties. The basic idea consists in feeding the amplifier field winding of the a-c reversible motor by a variable unidirectional current having a fundamental frequency (60 or 400 cycle) component and a d-c component producing an effective dynamic braking of the motor, preferably in such a way that the d-c component increases when the a-c component decreases, and vice versa.

There are various possibilities for designing of 2-stage magnetic servo amplifier circuits which are based upon the aforementioned principle. However, only two typical circuit arrangements will be

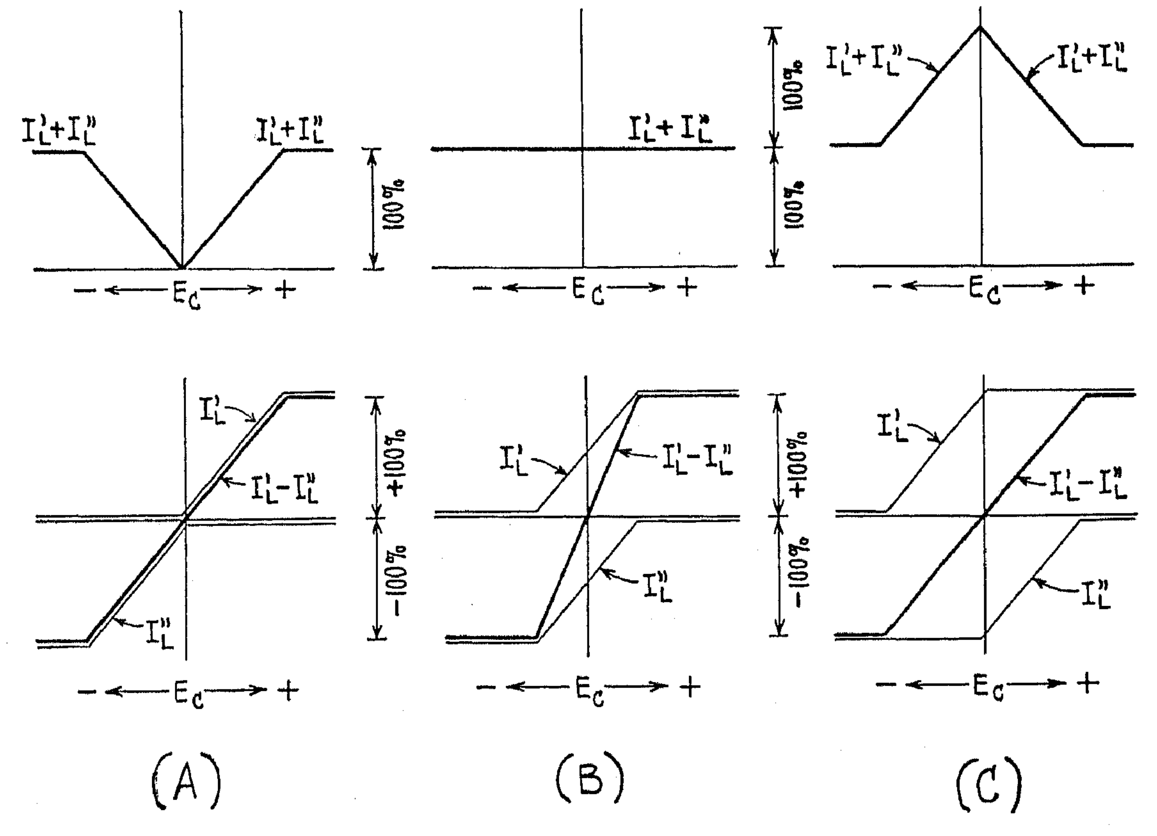


Figure 2. Three different operating conditions of the full-wave output-stage circuits, as shown in Figure 1

(A). There is no damping effect with no-signal conditions
(B). There is a constant damping effect
(C). There is a variable damping effect

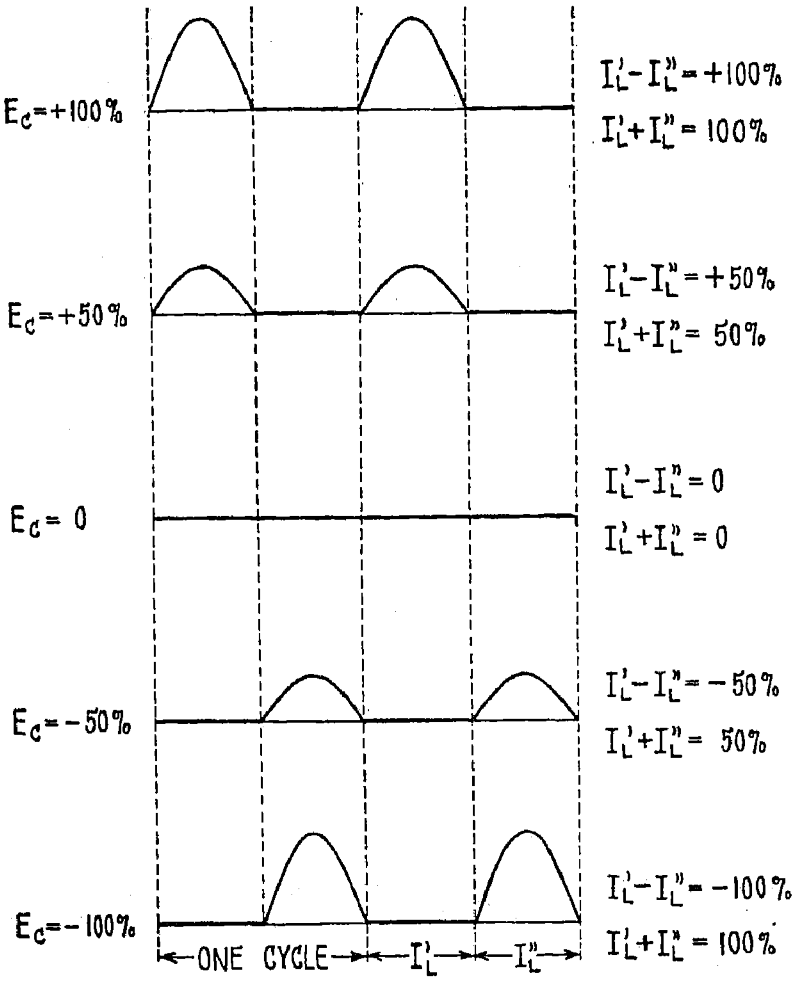


Figure 3 (left). The amplitudes of the half-cycle pulses I_L' and I_L'' for various values of control voltage E_C with operating conditions as shown in Figure 2(A)

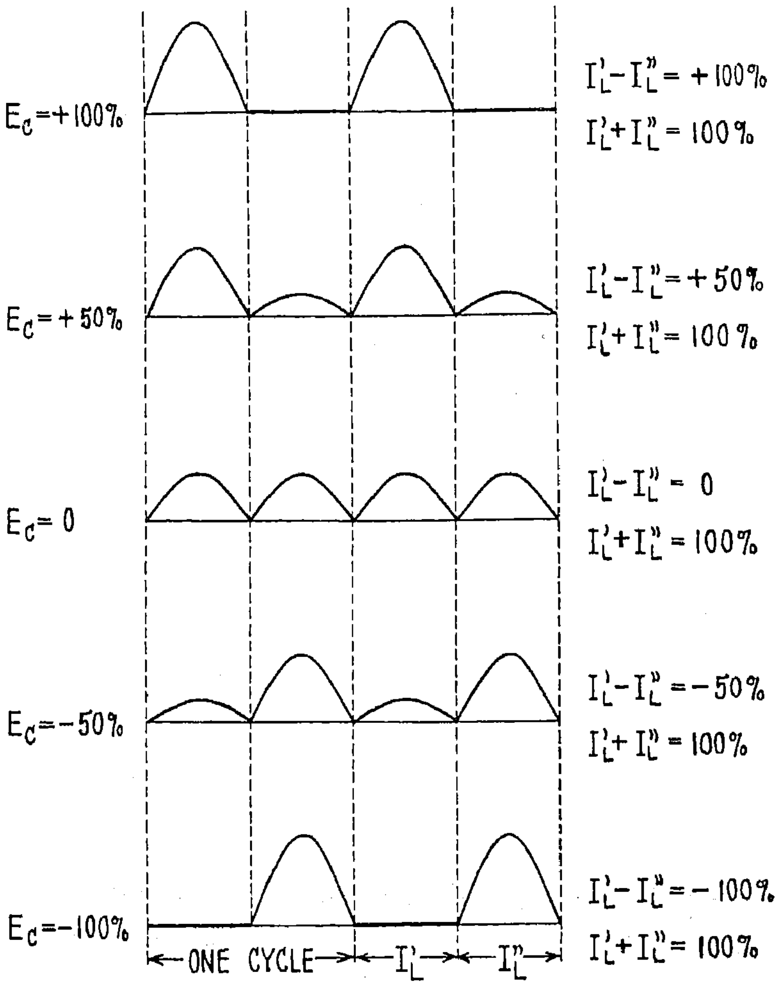


Figure 4 (right). The amplitudes of the half-cycle pulses I_L' and I_L'' for various values of control voltage E_C with operating conditions as shown in Figure 2(B)

treated in this paper to show the unique properties of these circuits which have inherent dynamic braking characteristics.

Damping Requirements

The conventional magnetic amplifiers of the balance detector type[6] are particularly suitable for operating those constructions of reversible induction motors which have inherent damping or special damping means.[5] However, some high-performance servomechanisms are operated by means of a 2-phase squirrel-cage motor embodying a comparatively high value of mechanical inertia and having low damping. In these cases, special auxiliary devices for producing effective damping must be employed.

When such an induction motor is to be controlled by means of a vacuum-tube amplifier, an effective damping of the motor itself will be obtained, because the direct current (quiescent current) flowing through the plate circuit of the output stage of this amplifier provides dynamic braking of the motor. However, with a magnetic amplifier the a-c output (load) current of the conventional types of full-wave push-pull circuits do not have a d-c component which may be used for producing dynamic braking of the motor.

Referring to the half-wave self-saturating push-pull type of magnetic amplifier a-c motor control circuits, as described by C. S. Hudson,[7] L. R. Crow,[8] and others, it is to be noted that the torque-producing a-c component and the associated d-c component of the unidirectional reversible motor control current will always increase or decrease at the same rate. Thus, there is no damping effect at all with no-signal conditions. This is generally not desirable when operating with those constructions of 2-phase induction motors which have very poor damping properties because there is no dynamic braking effect in the vicinity of the null point. However, these half-wave push-pull type circuits have proved to be very useful with a reversible a-c motor having inherent damping properties or special damping means.[5]

With regard to the application of derivative feedback in the field of magnetic servo amplifiers, it is important to note that some difficulty may be caused by the fact that the output (load) current of magnetic amplifiers—sometimes even with no-signal conditions—contains a very large a-c component superimposed on the d-c component, which is used for supplying the derivative feedback circuit.

The new full-wave push-pull circuits to be described now introduce the possibility of using very effective dynamic braking characteristics. These circuits contain:

1. A center-tapped power supply transformer T_P providing two equal voltages E_P', E_P'' which are proportional to the power supply voltage E_P (frequency f_P).
2. The load windings N_A', N_A'' of two equally rated saturable reactor elements, preferably having Orthonol tape cores.[9,10]
3. Two series-connected half-wave selenium rectifier sets S_A', S_A'' producing self-saturating effects.
4. A-c bias windings N_B', N_B'' with series resistors R_B', R_B''.
5. D-c control windings, Figure 1(A), or a-c control windings, Figure 1(B), N_C', N_C'' with a series resistor R_C.
6. The amplifier field winding W_L of the 2-phase reversible induction motor with shunt capacitor C_L.
7. The line field winding W_P of this motor with series capacitor C_P.

The basic full-wave output-stage circuits (Figure 1) differ from half-wave push-pull circuits[7,8] in that positive half-waves of current (I_L') flow through load winding N_A', and negative half-waves of current (I_L'') flow through load winding N_A''. In any case, the two control windings N_C' and N_C'', which may be connected with various types of d-c or a-c control circuits, are arranged so that push-pull action is obtained.

The a-c bias circuits $N_B'R_B'$ and $N_B''R_B''$ are provided to eliminate differences of the magnetic properties of the two saturable reactor elements, to bring the operating point of these elements to its most favorable position, and to produce special voltage-current characteristics of the magnetic amplifier. Another way for eliminating such differences consists in using a potentiometer-type resistor R_O to be adjusted so that the reversible motor will not move, if the control voltage E_C is zero.

Figure 2 illustrates three different operating conditions of the full-wave output-stage circuits of Figure 1 having inherent dynamic braking characteristics.

Figure 2(A): With $E_C = 0$ the two load

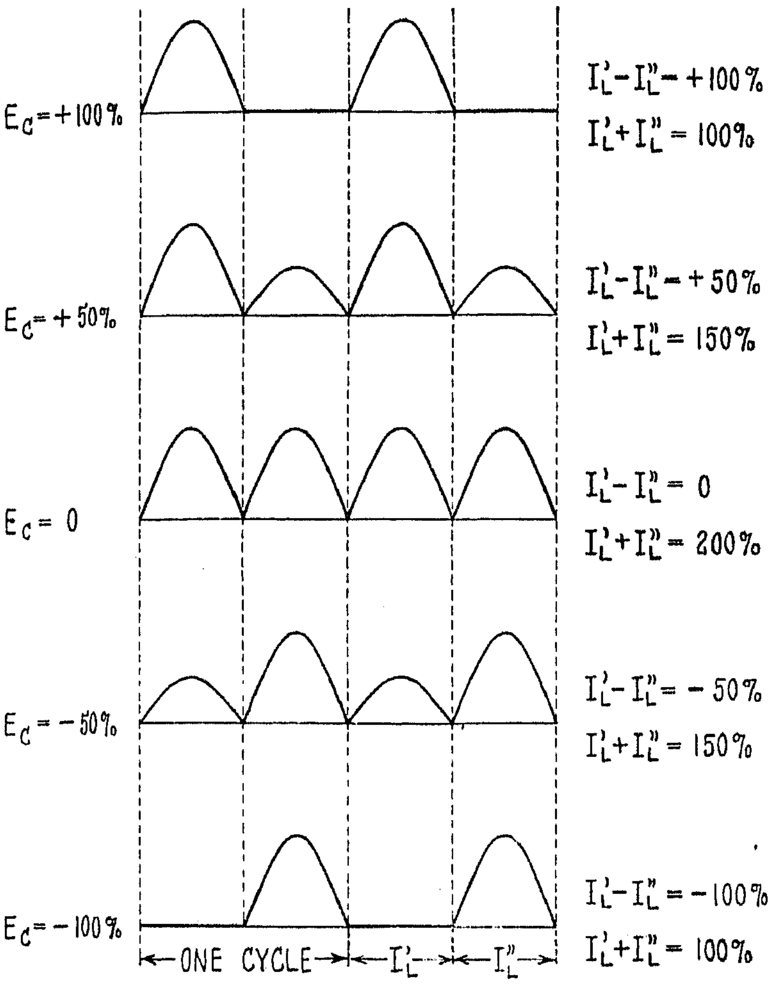
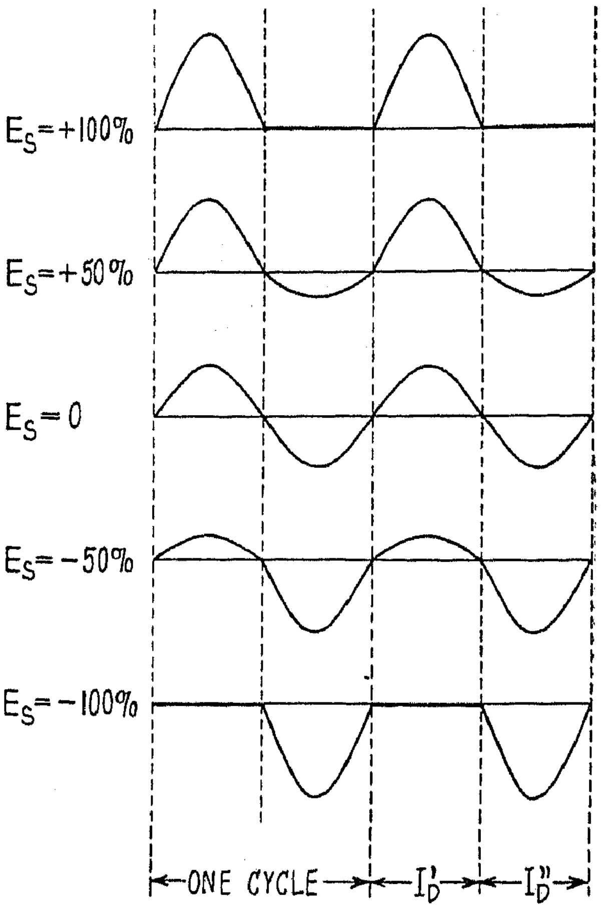


Figure 5 (left). The amplitudes of the half-cycle pulses I_L' and I_L'' for various values of control voltage E_C with operating conditions as shown in Figure 2(C)

Figure 7 (right). The actual amplitudes of the two half-cycle pulses I_D' and I_D'' for various values of control voltage E_S, corresponding to the operating conditions of the input-stage circuit as shown in Figure 6



(quiescent) currents I_L' and I_L'' are equal and very small, and there is practically no damping effect because the d-c component of $I_L'+I_L''$ is extremely small. However, $I_L'+I_L''$ and the corresponding damping effect increases, when E_C increases. This is generally not desirable, because the speed of the motor is decreased with higher values of error voltage E_C and because there is practically no damping effect in the vicinity of the null point.

Figure 2(B): With $E_C=0$ the two load (quiescent) currents I_L' and I_L'' are about 50 per cent of maximum load current, and there will be a large and constant damping effect because $I_L'+I_L''$ will have a constant magnitude over the entire range. These working conditions will give satisfactory results in many cases.

Figure 2(C): With $E_C=0$ the two load (quiescent) currents I_L' and I_L'' have about their maximum (100 per cent) values, and there will be a very high damping effect. It is to be noted, however, that the damping effect decreases if the error voltage E_C increases. This variable damping effect which is produced by the changing of the d-c component $I_L'+I_L''$ permits faster motor movement toward the null point and slower movement coincident with the lower values of error voltage E_C characterizing the close proximity of the null point. Thus, a very effective dynamic braking of the motor will be obtained in this way: When the a-c (60 or 400 cycle) component of load current $I_L=I_L'-I_L''$ disappears from the amplifier field winding W_L, the comparatively large d-c component of I_L tends to lock the rotor in the position assumed at the instant a-c excitation ceases.

Of course, it is possible in special cases to superimpose an additional d-c damping effect in the amplifier field winding (W_L) for further modification of the actual damping conditions.

Figures 3 to 5 show the amplitudes of the two half-cycle pulses I_L' and I_L'' for various values of control voltage E_C: +100 per cent, +50 per cent, 0, −50 per cent, and −100 per cent. The corresponding values $I_L'-I_L''$ of the a-c (60 or 400 cycle) component and the corresponding values $I_L'+I_L''$ of the d-c component also are indicated in this presentation which characterizes the three different operating conditions, as illustrated in Figure 2.

It is to be noted, however, that sinusoidal wave shape of the two half-cycle pulses I_L' and I_L'' has been assumed for purpose of illustrating the basic principle of push-pull full-wave output-stage circuits, as shown in Figure 1. The fact that the wave shape of saturable reactor type circuits is not sinusoidal does not change the operation principle of these circuits.

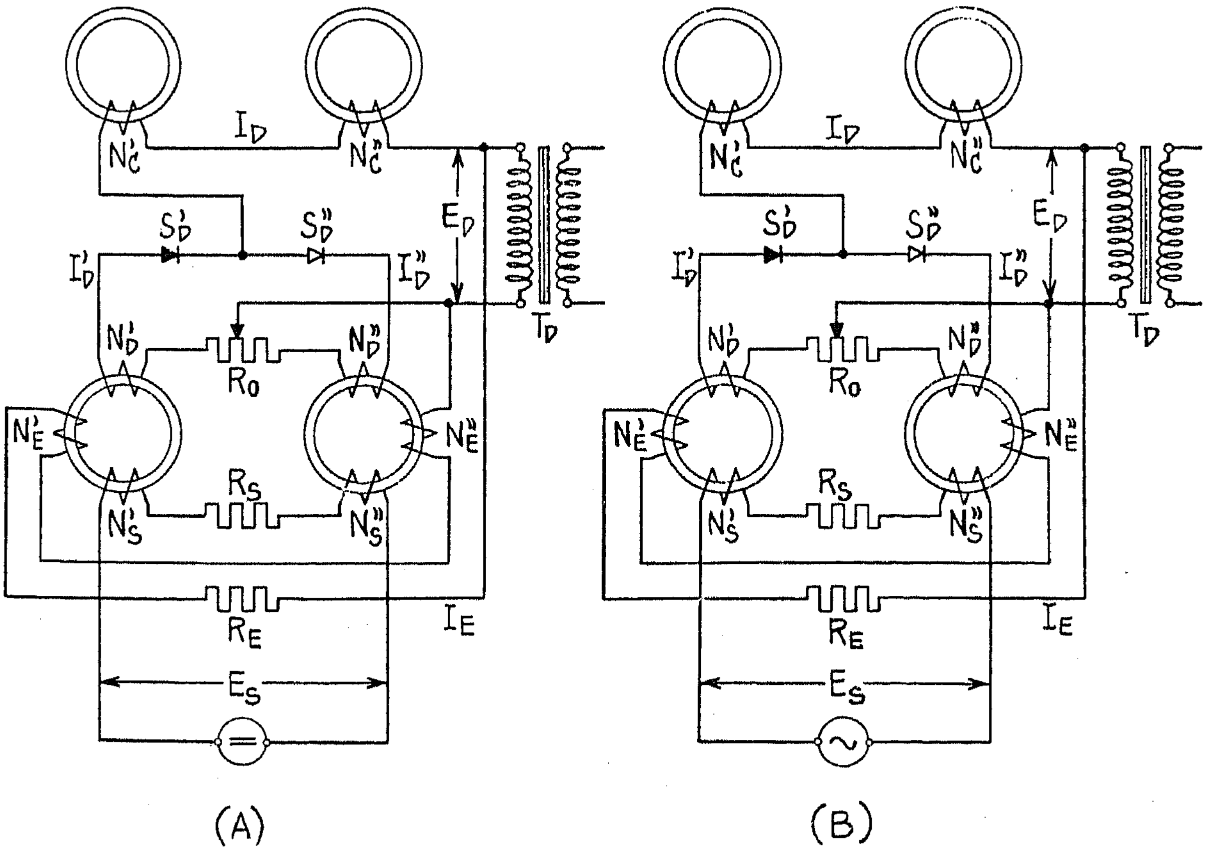


Figure 6. A new type of full-wave push-pull input-stage circuit containing two saturable reactor elements only, with d-c control windings (A) or with a-c control windings (B)

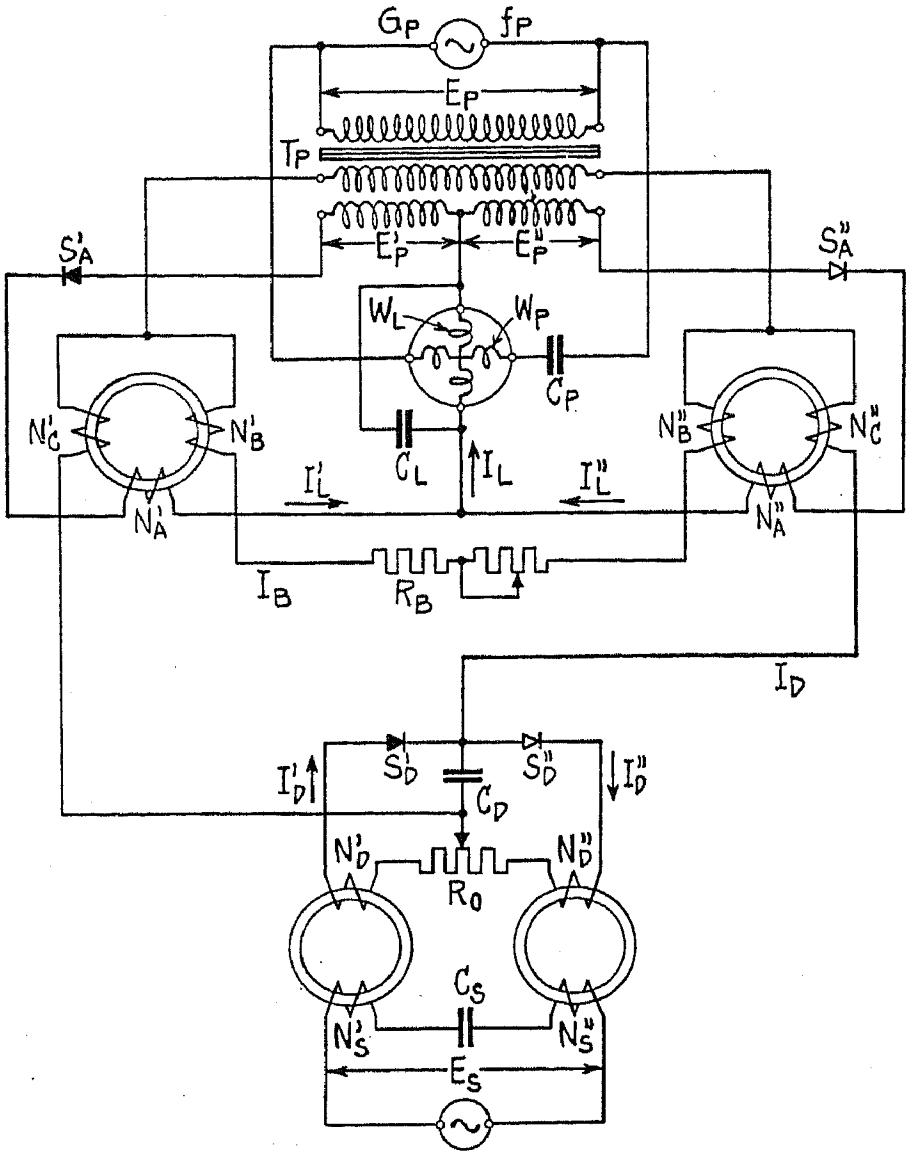


Figure 8. The new type of full-wave 2-stage magnetic servo amplifier circuit having a center-tapped power supply transformer (T_P)

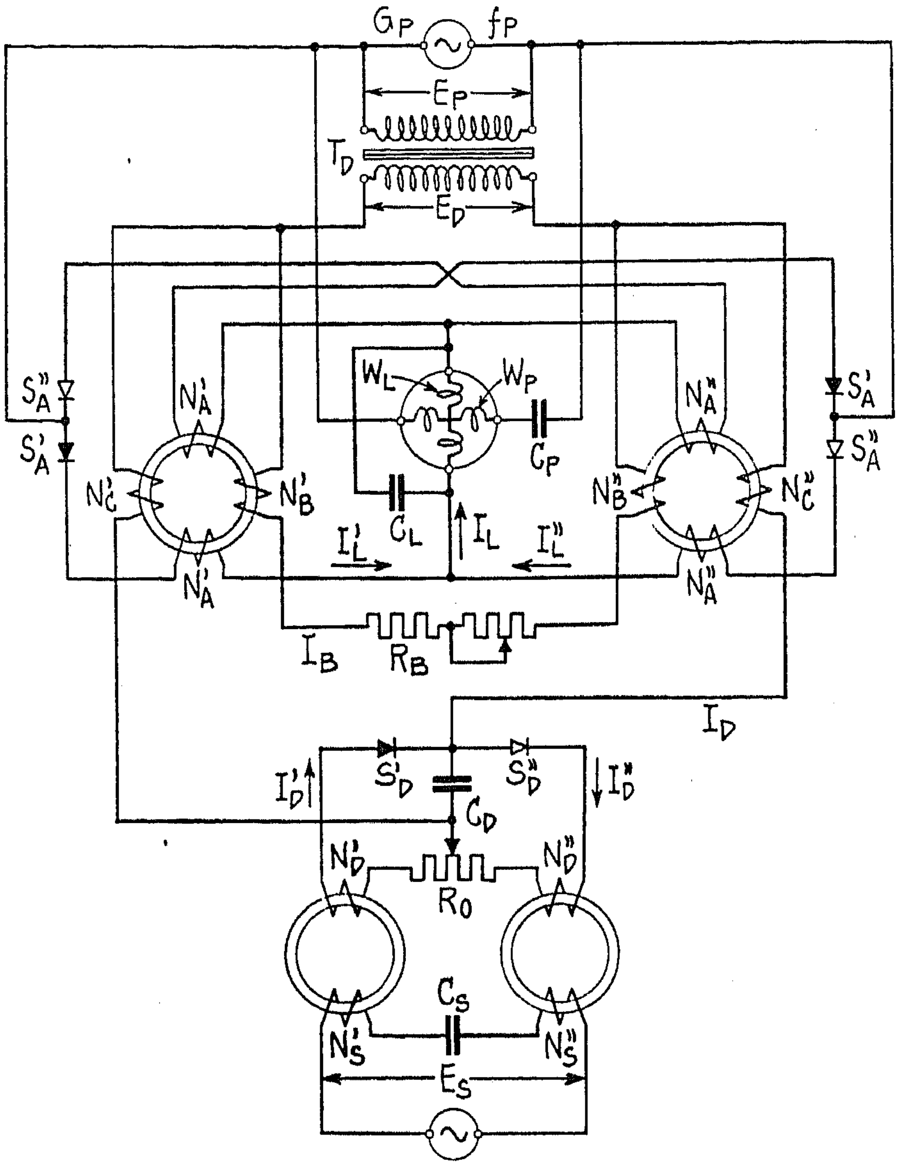


Figure 9. The new bridge type of full-wave 2-stage magnetic servo amplifier circuit having no power supply transformer

Basic Full-Wave Input-Stage Circuits

The various purposes of the input-stage circuit to be combined with the output-stage circuit, Figure 1, can be defined as follows:

1. To act as a push-pull type magnetic amplifier to produce a variable current, a function of the reversible error voltage E_S, for controlling the output-stage circuit.
2. To act as a phase discriminator (in a way similar to a phase-sensitive rectifier) so that actual direction of the resultant output current component $I_D = I_D' - I_D''$ will correspond to the actual phase displacement (0 degrees or 180 degrees) between error voltage E_S and power supply voltage E_P.
3. To act as a voltage-sensitive bias element to the output-stage circuit for introducing special voltage-current characteristics.
4. To act as a decoupling device in such a way that no appreciable alternating voltage will be induced in the synchro-transformer a-c control circuit having a total impedance of about 10,000 ohms.

The designer, when outlining the circuitry to be used in this special case, faces the following problem: The output currents of the push-pull input-stage circuit, preferably having two saturable reactor elements only, must provide

1. A bias component which is a function of the actual magnitude of power supply voltage E_P.
2. A reversible control component, which is a function of the actual error voltage E_S.

Furthermore, it is to be noted that—in contrast to conventional d-c control circuitry—comparatively large voltages of fundamental frequency will be induced in the two control windings N_C' and N_C'' of the output-stage circuit, Figure 1, even when the actual turn ratio $N_C'/N_A' = N_C''/N_A''$ has small value (about 1/40).

Figure 6 shows as an example a suitable 2-core type of push-pull self-saturating circuit which may be controlled either by direct or alternating current. This circuit contains:

1. The load windings N_D', N_D'' of two equally rated saturable reactor elements, preferably having Mumetal tape cores.
2. Two series-connected half-wave selenium rectifier sets S_D', S_D'' producing self-saturating effects.
3. A-c bias windings N_E', N_E'' with a series resistor R_E.
4. D-c or a-c control windings N_S', N_S'' with a series resistor R_S.
5. The control windings N_C , N_C'' of the push-pull output-stage circuit, as shown in Figure 1(A).
6. A potentiometer-type resistor R_O for zero adjustment.
7. A small transformer T_D (for example, 115/20 volts).

The a-c controlled input-stage circuit, Figure 6(B), will be used in the 2-stage magnetic servo amplifier operating in connection with a synchro-control transformer arrangement. In other cases, however, the d-c controlled input-stage circuit, Figure 6(A), may be valuable, particularly for applications in the field of d-c measurements using thermocouples, photoelectric cells, and so forth.

Figure 7 gives an idealized picture of the amplitudes of the two half-cycle pulses I_D' and I_D'' for various values of control voltage E_S: +100 per cent, +50 per cent, 0, −50 per cent, −100 per cent. Considering the fundamental circuit, as shown in Figure 6, it is evident that alternating current can pass freely through this circuit even though each dry-disc rectifier element is a unidirectional device. With $E_S = 0$, the impedance of the two saturable reactor elements will be equal, and the two half-cycle pulses I_D' and I_D'' will have the same magnitude. Under these conditions, the two unidirectional currents I_D', I_D'' flowing through the control windings N_C', N_C'' of the output-stage circuit will produce a voltage-sensitive bias effect and the desired voltage current characteristics. Since the two half-cycle pulses I_D' and I_D'' are equal in magnitude and identical in wave shape, there is no control current difference $I_D = I_D' - I_D''$ present in the output-stage control windings N_C', N_C'' to run the a-c reversible motor.

However, if a voltage E_S is applied to the control circuit, the impedances of the two saturable reactor elements will have different values, the two half-cycle pulses I_D' and I_D'' will have different magnitudes, and there will be a control current difference $I_D = I_D' - I_D''$ which is a function of error voltage E_S. Actual direction of I_D will correspond to the actual phase displacement (0 degrees or 180 degrees) between error voltage E_S and reference voltage E_D, which is proportional to the 60- or 400-cycle power supply voltage E_P. The control current difference $I_D = I_D' - I_D''$ will operate the a-c reversible motor in such a way that the direction of movement will correspond to the actual phase displacement (0 degrees or 180 degrees) between the voltages E_S and E_D.

Full-Wave 2-Stage Circuits

Correct and efficient operation of the complete 2-stage circuit will be obtained only when the a-c (60 or 400 cycle) components of the currents flowing through the various parts of this circuit have a proper phase relationship. The following two examples show a simple way for obtaining proper phase displacement by means of four capacitors so that the most favorable working conditions will be secured. The final 2-stage circuits are results of a special experimental study, and the optimum values of the capacitors. which are not critical, have been determined empirically.

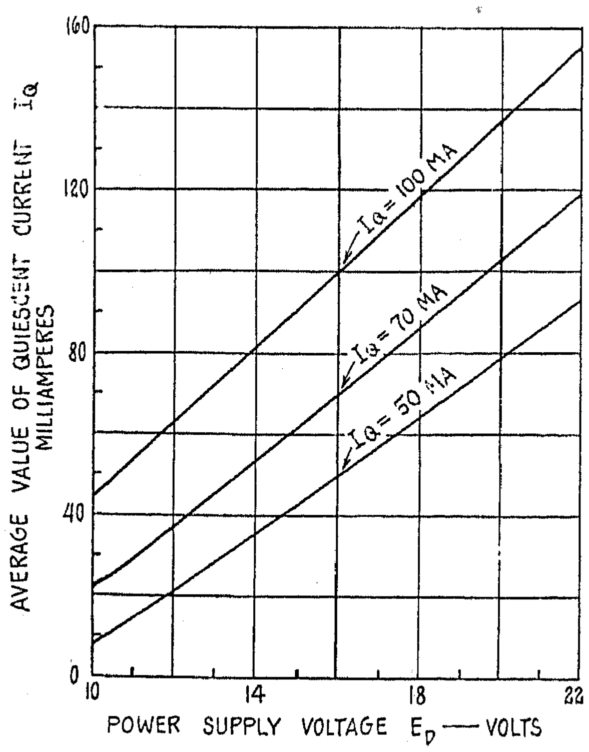


Figure 10. Quiescent current I_Q as a function of power supply voltage E_D, if constant biasing ampere-turns are supplied to the cores of the input-stage circuit as shown in Figure 6

Figure 8 shows a full-wave 2-stage circuit which contains:

1. The output-stage circuit, as shown in Figure 1.
2. The input-stage circuit, as shown in Figure 6.
3. The power supply transformer T_P having a center-tapped secondary winding (voltages E_P', E_P'') and an additional secondary winding supplying the input-stage circuit.

Proper phase relationship of the currents flowing through the various parts of this 2-stage circuit and maximum torque conditions of the 2-phase motor will be obtained by means of the capacitors C_P, C_L, C_D, and C_S.

It is to be noted that the actual values of output-stage quiescent currents I_L', I_L'' can easily be controlled by adjusting the variable a-c bias resistor R_B.

The bridge-type output-stage circuit, Figure 9, makes it possible to eliminate the power supply transformer (T_P in Figure 8) and to reduce size and weight of the complete magnetic servo amplifier by dividing the load windings N_A', N_A'' in two equally rated parts which are insulated from each other. It is to be noted that the total number of dry-disc rectifier cells remains unchanged, so that, for example, 4 by 12 cells may be used in the bridge type circuit (Figure 9), whereas 2 by 24 cells have to be provided in the transformer type circuit (Figure 8).

It is to be noted that application of the small power supply transformer T_D (for example, 115/20 volts) makes it possible to reduce the number of cells of the rectifiers S_D', S_D'' and to reduce the number of turns of input-stage load windings N_D', N_D'' and output-stage control windings N_C', N_C''. The transformer T_D, however, also may be eliminated by properly varying these circuit components.

Furthermore, in any case, the two input-stage control windings N_S' and N_S'', each having its own series capacitor, may be connected in parallel to the control voltage E_S of the synchro control transformer to obtain perfect symmetry of the transfer characteristic $I_L = f(E_S)$ by correct rating of these two series capacitors.

Referring to the method for obtaining perfect zero adjustment ($I_L = I_L' - I_L'' = 0$, if $E_S = 0$), it is to be noted that this can be achieved easily by adjusting the potentiometer—type resistor R_0 (Figures 8 and 9) which controls the actual value of current ratio I_D'/I_D'' of the input-stage circuit.

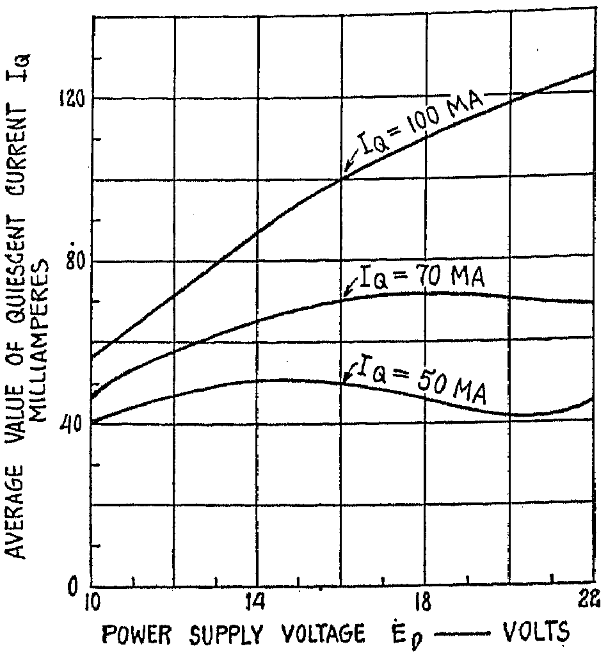


Figure 11. Quiescent current I_Q as a function of power supply voltage E_D, if variable biasing ampere-turns are supplied to the cores of the input-stage circuit as shown in Figure 6

Results of Measurements

To get some information regarding the actual performance of the new full-wave push-pull type circuits, various measurements on a representative 2-stage design have been made. The results of these measurements show that this magnetic servo amplifier has unique performance characteristics when nickel-iron-alloy tape cores with multilayer toroidal windings are used in the actual circuits.

Characteristics of the Input-Stage Circuit

This circuit, Figure 6(B), contains two Mumetal 2-mil tape cores (outside diameter: 2 inches, inside diameter: $1^5/_8$ inches, tape width: 0.5 inch) having multilayer toroidal windings ($N_D = 2{,}000$, $N_E = 50$, $N_S = 200$, each with number 28 wire size). The magnetic circuit has an effective area of 0.084 square inch = 0.54 square centimeter, and a mean length of 5.7 inches = 14.5 centimeters.

Selenium rectifier sets are used in such a way that the maximum value of reverse voltage on each rectifier cell does not exceed about 10 volts, to obtain favorable working conditions regarding the reverse resistances and the corresponding reverse-current components of the self-saturating circuit.

Referring to bias circuitry for magnetic amplifiers, it is important to stress that there are two ways of injecting into the core a magnetic biasing flux:

1. By supplying the core with constant biasing ampere-turns.
2. By supplying the core with variable biasing ampere-turns, preferably being pro-

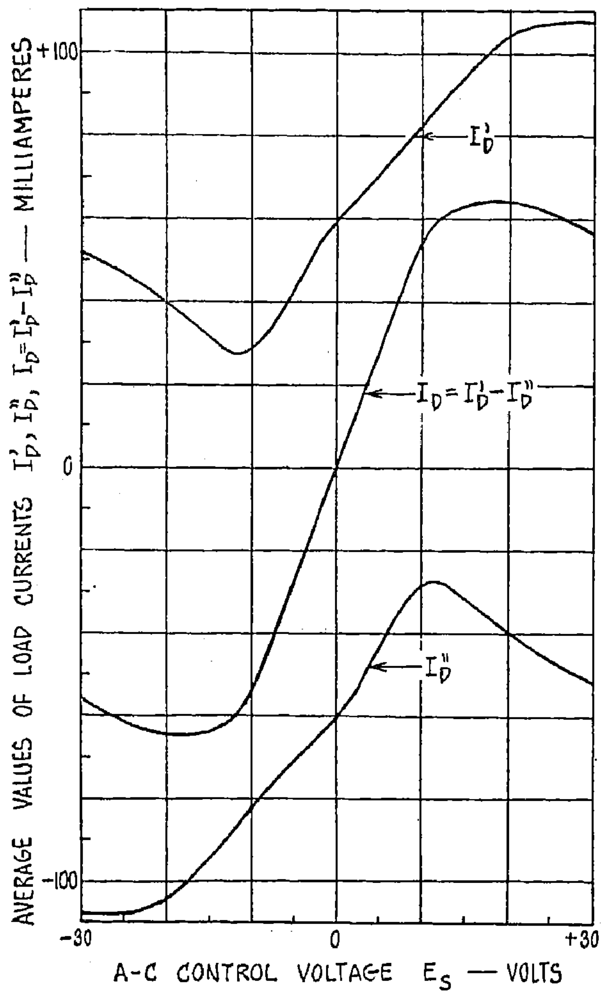


Figure 12. Load currents I_D' and I_D'' and control current difference $I_D=I_D'-I_D''$ as a function of the alternating control voltage E_S when the input-stage circuit, Figure 6(B), is used

portional to the power supply voltage of the magnetic amplifier.

Ten years ago the author[11] suggested designing magnetic servo amplifiers utilizing external feedback or self-saturation in such a way that the magnitude of the quiescent current is substantially independent of changes in a-c supply voltage. This was accomplished by applying special voltage-sensitive bias circuits which introduce inherent constant-current characteristics in the magnetic amplifier.

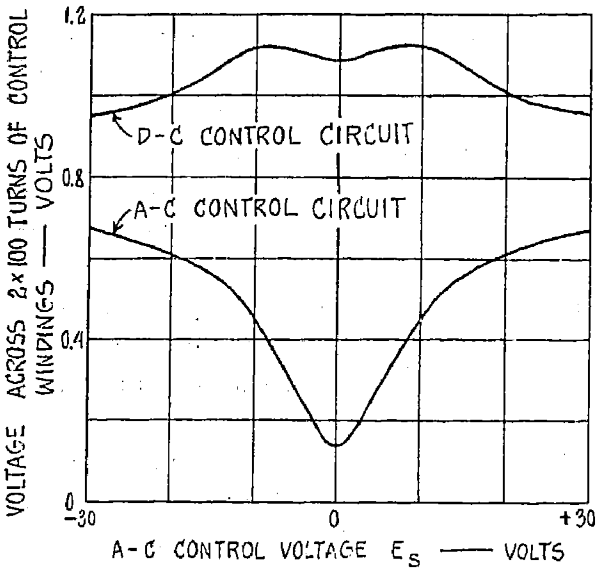


Figure 13. Actual voltage of fundamental frequency which is induced in series-connected control windings N_B' and N_B'', both for a d-c control circuit, Figure 6(A), and for an a-c control circuit, Figure 6(B)

Figure 10 shows the quiescent current I_Q as a function of power supply voltage E_D, if constant biasing ampere-turns are supplied to the cores of the input-stage circuit, as shown in Figure 6(B). The actual quiescent current value $I_Q=50$, 70, or 100 milliamperes corresponding to the power supply voltage $E_D=16$ volts is used as a parameter. Figure 11 shows the voltage-current characteristic $I_Q=f(E_D)$ of the same circuit, if variable biasing ampere-turns being proportional to E_D are used.

The results of these measurements using either constant or voltage-sensitive bias circuitary, as illustrated in Figures 10 and 11, show in a dramatic way the progress which is of paramount importance for successful development of magnetic servo amplifiers which must have an extremely small drift rate, so that the performance of the servomechanism is not appreciably influenced by changes of from ±5 to 10 per cent in power-supply voltage. It is interesting to note that special voltage-current characteristics, as shown in Figure 11, are due to the inherent negative-feedback properties of voltage-sensitive bias circuits: When the power-supply voltage E_D increases, then the bias current I_E (Figure 6) increases correspondingly so that the quiescent current I_Q will be controlled as to have a constant magnitude. Thus, the bias windings N_E', N_E'' carrying $I_E=\text{const}\times E_D$ act as additional control winding,s to make I_Q practically independent of changes within the limits from ±5 to 10 per cent in voltage E_D.

Figure 12 shows the load currents I_D', I_D'' and $I_D=I_D'-I_D''$ as a function of the alternating control voltage E_S when the a-c controlled input-stage circuit, Figure 6(B), is used. Figure 13 shows the actual voltage of fundamental frequency which is induced in series-connected windings N_S', N_S'' of a d-c control circuit, Figure 6(A), and an a-c control circuit, Figure 6(B).

Characteristics of the Output-Stage Circuit

This circuit (Figure 1) contains two Orthonol 2-mil tape cores (outside diameter: 2 inches, inside diameter: $1^5/_8$ inches, tape width: 1.0 inch) having multilayer toroidal windings ($N_A=4{,}000$, $N_B=100$, $N_C=100$, each with number 31 wire size). The magnetic circuit of each core has an effective area of 0.168 square inch = 1.08 square centimeter, and a mean length of 5.7 inches = 14.5 centimeters.

Two selenium rectifier sets (each having 24 cells) are used in connection with a center-tapped power supply transformer T_P providing two equal secondary voltages $E_P'=E_P''=150$ volts, to obtain favorable working conditions regarding the reverse resistances and the corresponding reverse-current components of the self-saturating circuit.

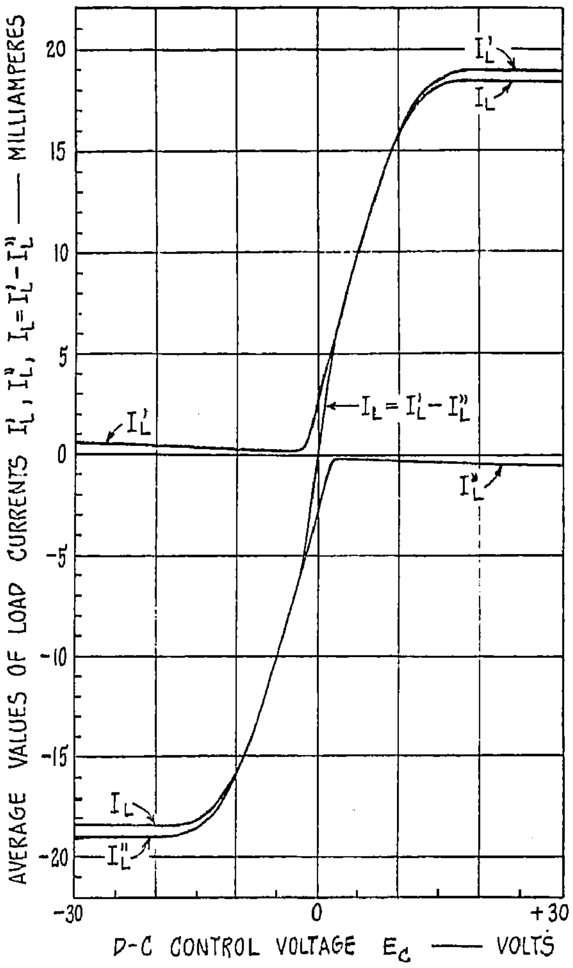


Figure 14. Load currents I_L', I_L'', and $I_L=I_L'-I_L''$ as a function of d-c control voltage E_C with actual quiescent current value $I_Q=3$ milliamperes

Figures 14 to 16 show the measured values of load currents I_L', I_L'', and $I_L=I_L'-I_L''$ as a function of d-c control voltage E_C with $R_C=1{,}000$ ohms; Figure 1 (A). The actual quiescent current value $I_Q=3$, 10, or 17 milliamperes corresponding to the power-supply voltage $E_P'=E_P''=150$ volts is used as a parameter.

In Figure 14, $I_Q=3$ milliamperes; in Figure 15, $I_Q=10$ milliamperes; and in Figure 16: $I_Q=17$ milliamperes.

Figure 17 shows the measured average value of the d-c component $I_{DC}=I_L'+I_L''$ of load current I_L as a function of d-c control voltage E_C with quiescent current I_Q (3, 10, or 17 milliamperes with $E_P'=E_P''=150$ volts) as a parameter. Figure 18 shows the actual voltages of fundamental frequency induced in additional test windings (each having 100 turns) which correspond to the control windings N_C', N_C'' of the two saturable reactor elements.

Characteristics of the 2-Stage Circuit

This circuit (Figure 9) has been investigated in connection with a standard type of servomechanism which is operated

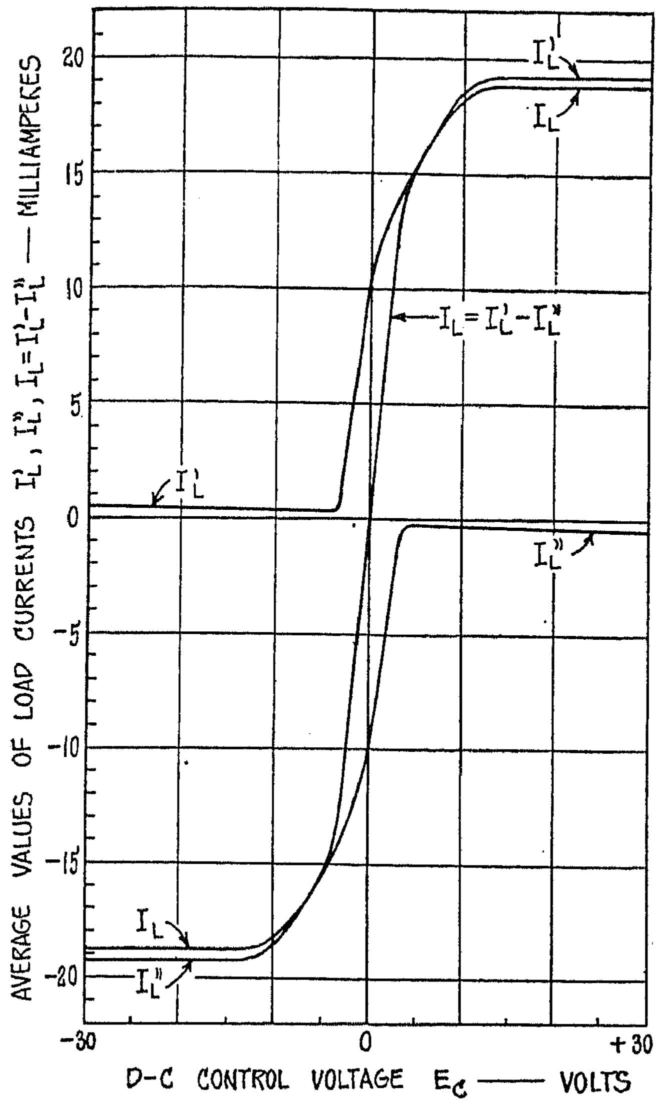


Figure 15 (left). Load currents I_L', I_L'', and $I_L=I_L'-I_L''$ as a function of d-c control voltage E_C with actual quiescent current value $I_Q=10$ milliamperes

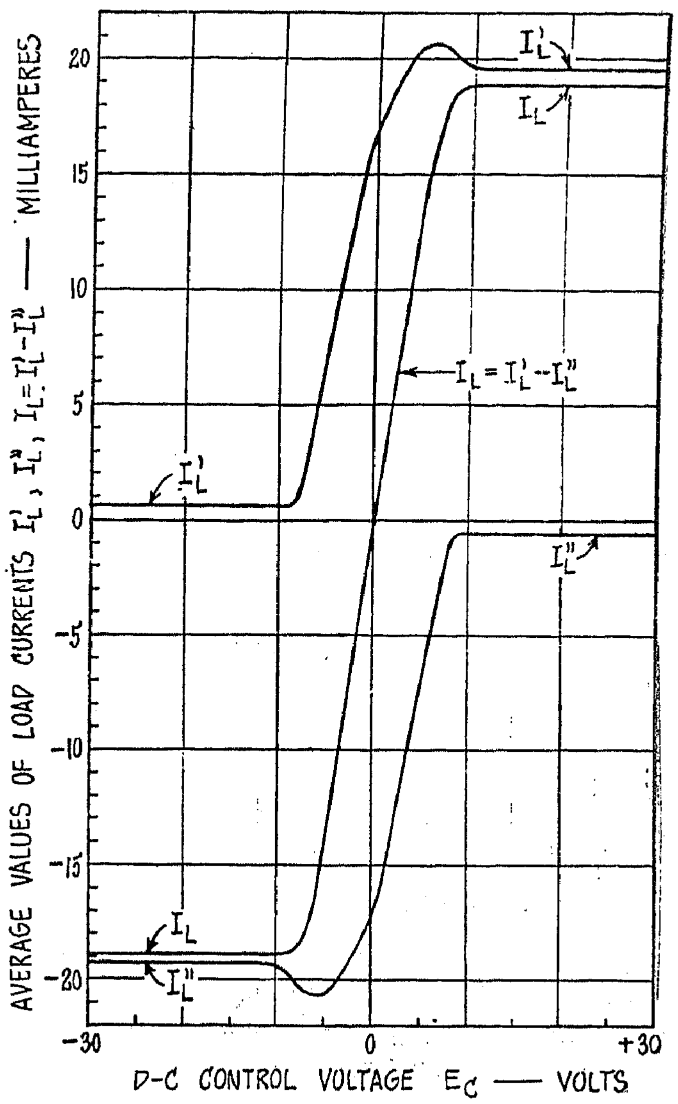


Figure 16 (right). Load currents I_L', I_L'', and $I_L=I_L'-I_L''$ as a function of d-c control voltage E_C with actual quiescent current $I_Q=17$ milliamperes

by means of a 3.5-watt 2-phase reversible induction motor having very poor damping properties and no additional damping means.

Figure 19 shows the resultant a-c load current $I_L=I_L'-I_L''$ of the output-stage circuit as a function of the a-c control voltage E_S of the input-stage circuit. Figure 20 shows the actual rotor speed of the 2-phase motor under no-load conditions as a function of the a-c control voltage E_S of the input-stage circuit. Referring to Figures 19 and 20, it is evident that an error voltage of about ±10 volts will give full output power of the magnetic amplifier.

As a result of various measurements it was learned that the static error is about 0.05 degree, while the drift error is about 0.1 to 0.2 degree.

Special Problems

A comparison between the conventional type 2-stage circuits having generally

1. eight saturable reactor elements (four elements in each stage),
2. four pairs of dry-disc rectifier elements (two pairs in each stage), and
3. an additional phase-sensitive rectifier circuit

and the new type of 2-stage circuits (Figures 8 and 9) having

1. four saturable reactor elements only (two elements in each stage),
2. two pairs of dry-disc rectifier elements only (one pair in each stage), and
3. no additional phase-sensitive rectifier circuit

shows that considerable economy will be achieved by the use of half the amount of basic components (saturable reactors and dry-disc rectifier elements), particularly with respect to ease of manufacture.

Referring to some special problems, which are involved in magnetic amplifier push-pull circuitry using two saturable reactor elements in each stage only, it is interesting to stress the following facts:

1. Those d-c controlled magnetic amplifier push-pull circuits, containing four saturable reactor elements in each stage,[6] have the unique feature that the fundamental frequency voltages, which are induced in the four series-connected control windings, are always cancelling each other completely, not only with no-signal

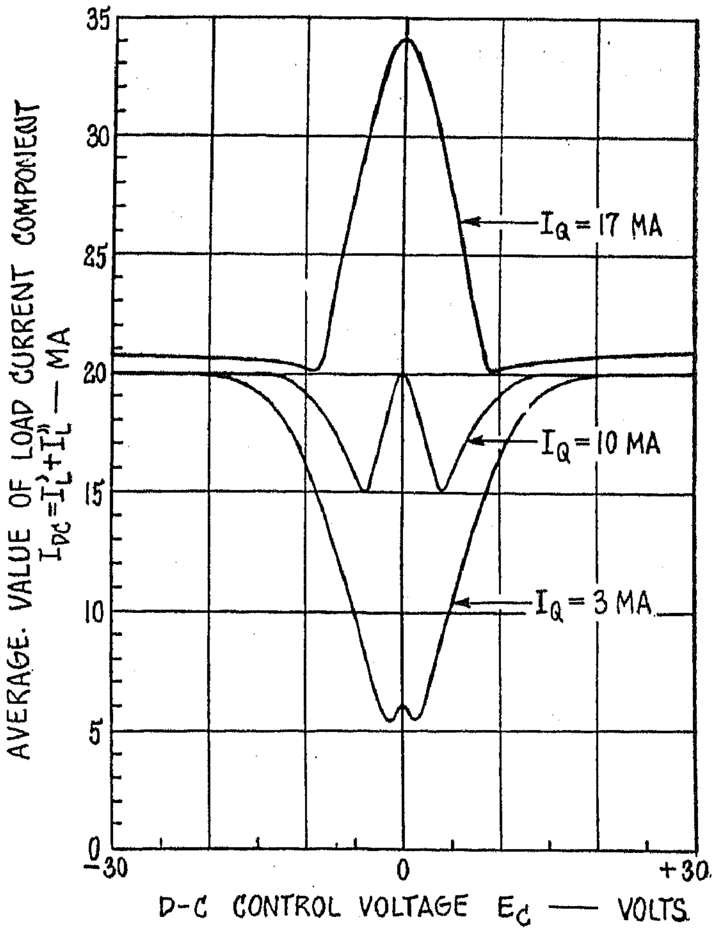


Figure 17. Average value of the d-c component $I_{DC}=I_L'+I_L''$ of load current I_L as a function of d-c control voltage E_C with quiescent current value I_Q as a parameter

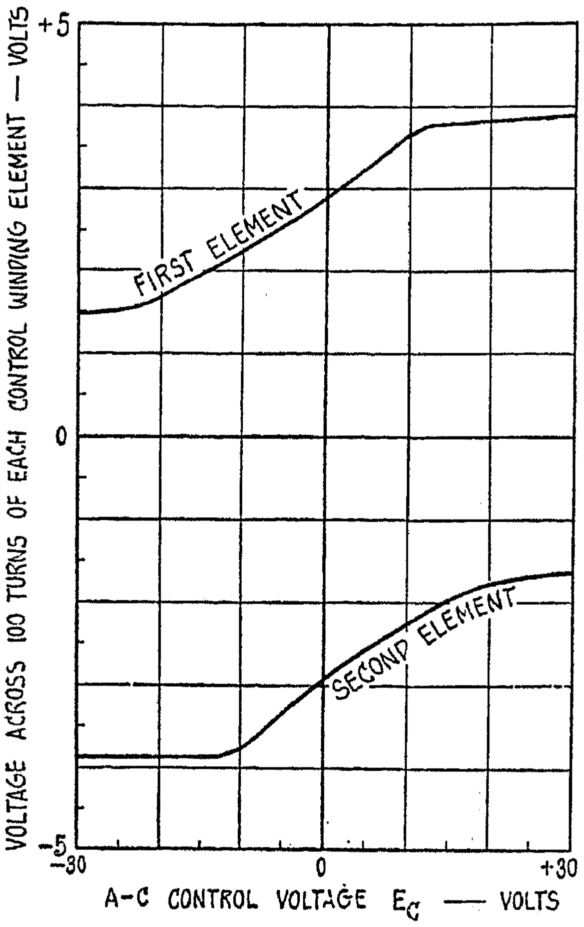


Figure 18. Actual voltages of fundamental frequency which are induced in the two control windings N_C' and N_C'' of the output-stage circuit (Figure 1)

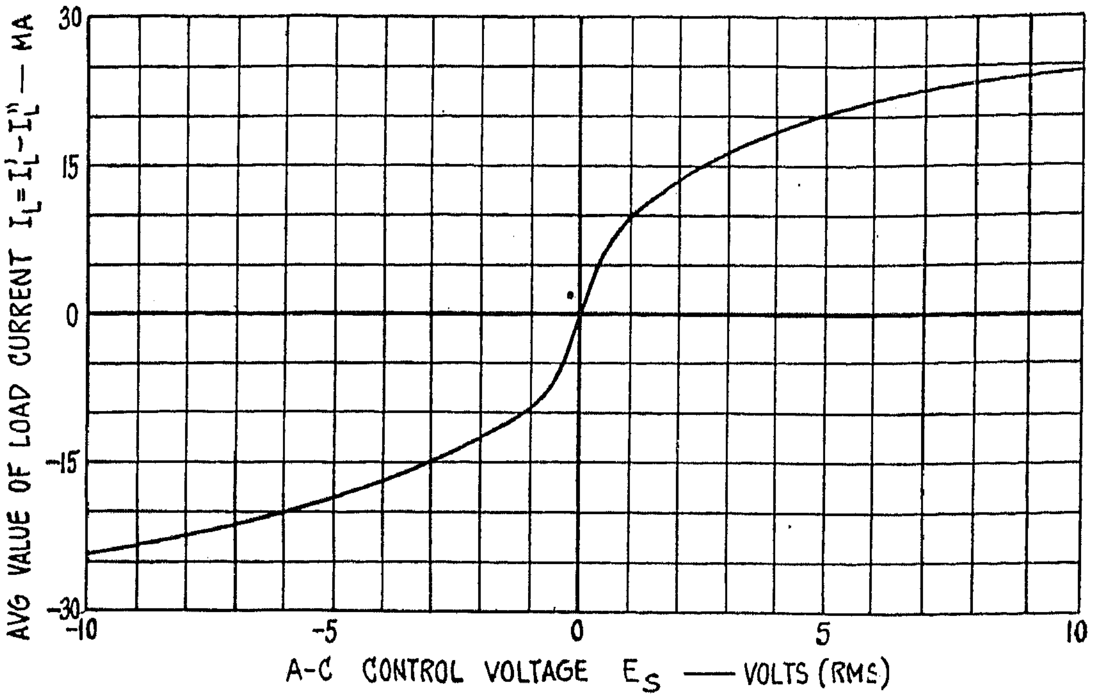


Figure 19. Resultant a-c load current $I_L = I_L' - I_L''$ of the output-stage circuit as a function of the a-c control voltage E_S of the input-stage circuit

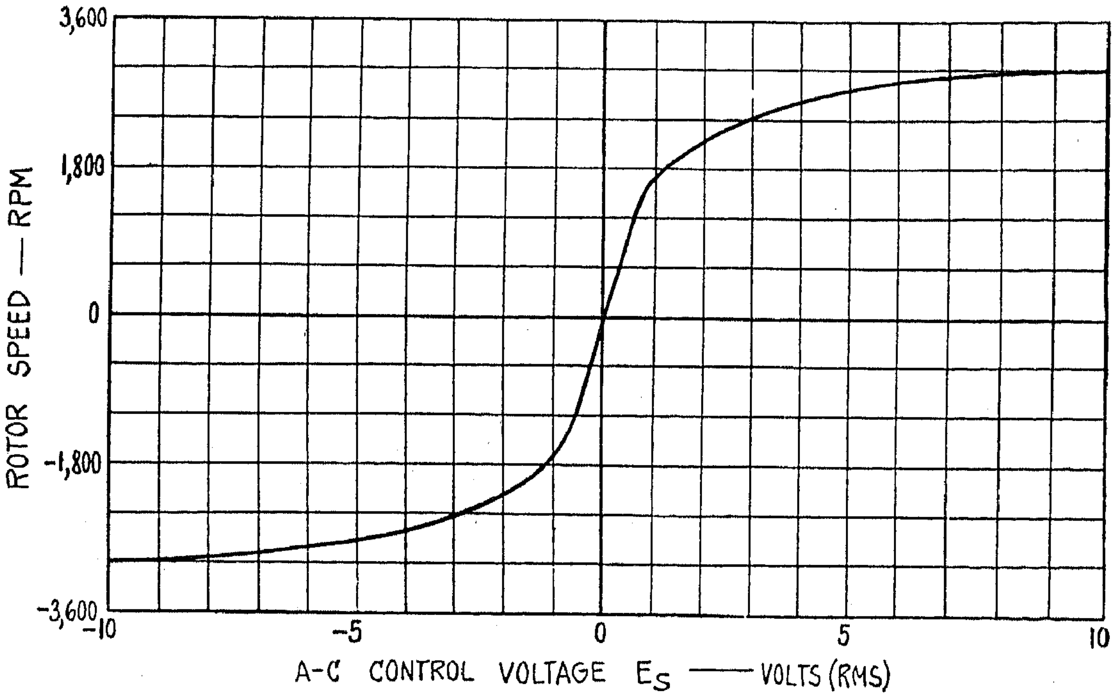


Figure 20. Actual rotor speed of the 2-phase induction motor under no-load conditions as a function of the a-c control voltage E_S of the input-stage circuit

conditions, but even when the maximum value of control voltage is applied to the amplifier.

However, any magnetic amplifier push-pull circuit containing two saturable reactor elements in each stage only, as described in this paper, has the typical property that the fundamental frequency voltages, which are induced in the two series-connected control windings, will cancel each other for no-signal conditions only; otherwise fundamental frequency voltages having considerable magnitudes may be induced in these control windings. Thus, the actual turn ratio $N_C'/N_A' = N_C''/N_A''$ (Figure 1) and $N_S'/N_D' = N_S''/N_D''$ (Figure 6) must have a very small value (about 0.01 to 0.1) to avoid serious difficulties.

2. In principle, it will be necessary to eliminate those effects which are caused by the actual differences of the magnetic properties of the saturable reactor elements, and by the actual differences of the dry-disc rectifier characteristics. Generally, this may be accomplished by providing special a-c bias circuits (Figures 1, 6, 8, and 9) containing variable resistors, so that the operating conditions of each saturable reactor self-saturating circuit can be adjusted individually. This is particularly important to secure a reliable manufacturing process of magnetic servo amplifiers.

It was learned, however, that the 2-stage full-wave circuit, as shown in Figures 8 and 9, can be adjusted easily by using one potentiometer resistor (R_o) only. This fact means a considerable simplification of the complete circuit for universal application.

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No Discussion

An Improved Magnetic Servo Amplifier

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OF THE potential applications of magnetic amplifiers, one of the most promising is in the field of automatic feedback controls. In such systems, reliability of operation is frequently as important in design considerations as the operational characteristics themselves. For this reason, the replacement of relatively short-lived vacuum tube amplifiers with long-lived, rugged, and reliable magnetic amplifiers is an extremely desirable thing provided the minimum operational characteristics can be met using the magnetic amplifier.

In the past, one of the major difficulties in obtaining high performance from a magnetic amplifier servo system has been the slow speed of response of the amplifier itself. Although the magnetic amplifier is a carrier-type system and as such has a minimum response time of 1 cycle of the supply or carrier frequency, this minimum time is usually not obtained in conventional circuitry.[1] As a result, very serious stability problems are encountered if such an amplifier is placed in a high performance servo loop. Stabilizing or corrective networks, such as error rate, velocity error, acceleration, and so forth, as conventionally used in vacuum tube servo amplifier design to obtain improved operational performance and stability, cannot be used with any degree of success when directly applied to magnetic amplifier circuitry. This is due in large part to the inherent low impedance of the circuits involved.

Despite these drawbacks, the increase in reliability, ruggedness, and so forth, obtained by using magnetic amplifiers has made their use not only desirable but frequently necessary in many applications.

The magnetic amplifier and servo design group of the Magnetics Division of the Naval Ordnance Laboratory has been interested primarily in designing and developing magnetic amplifiers suitable for use in high-performance instrument-type servos for remote positioning applications. An instrument servo is defined as one in which the load inertia and friction, reflected through the gear train to the motor shaft, is negligible compared to inertia and friction of the motor itself. This report covers the development of a new type magnetic amplifier circuitry and gives design and performance data of typical servos of the aforementioned type using this new circuitry.

The Conventional Magnetic Servo Amplifier

Instrument servos of the type being considered generally use a 2-phase induction motor as the power drive and synchro units for error detection. It is thus necessary that the servo amplifier be capable of receiving a phase reversible a-c signal and delivering a phase reversible a-c output, the magnitude being proportional to the input magnitude and the phase being determined by the input phase. A simple single-stage magnetic servo amplifier using conventional circuitry is shown in Figure 1. This circuit which delivers a full-wave phase sensitive output is based on the parallel self-saturating circuitry of Logan.[2] Variations and modifications of this circuitry have in the past formed the basis of most magnetic servo amplifier designs.

This basic circuitry, however, has certain drawbacks for use in a servo amplifier. For the most part, these drawbacks constitute features which contribute to the over-all time constant or speed of response of the amplifier. The magnetic amplifier is a carrier-type amplifier and as such will have inherently a minimum response time of 1 cycle of the power or carrier frequency. However, using this conventional circuitry, this minimum response time is usually not achieved. Instead, a delay of from 2 to 20, or more, cycles is to be expected. A major factor is the L/R time constant of the control circuit. Referring to Figure 1, it is evident that the control is established through series-connected windings which encompass all four reactors.

The inductive impedance Z_L of any given control winding may be reduced greatly when the reactor upon which it is wound goes into saturation. However, at no time during the operating cycle will this occur on more than two of the control windings simultaneously. Therefore, the total Z_L of two of the windings will be the minimum ever presented to the over-all control circuit during a given operating cycle. The ratio L/R may be, and conventionally is, decreased by addition of a series resistance in the control circuit (R_C, Figure 1). In this manner, improved speed of response may be obtained. The inclusion of R_C, however, may decrease the over-all gain of the amplifier tremendously. Its inclusion also requires that control turns be the major factor in building up gain for the system since its presence tends to limit the control current. To increase control ampere turns with the current limited, requires additional control turns. It is thus not uncommon to find control windings running well into the thousands of turns in amplifiers of such design. Certainly the more control turns per reactor, the higher the control circuit Z_L. This in turn means a higher R_C if the L/R time constant is to remain unchanged. It is evident, therefore, that speed of response and gain are not independent variables.[3,4] Unfortunately, the gain requirements for most servo amplifiers are such that they cannot be met without seriously increasing the speed of response under these conditions.

In addition to the control circuit time constants, load and, if present, bias circuit time constants may be detrimental. These are, however, usually small compared to the control L/R and are generally neglected. Another possible source of delay is the presence of circulating currents within the amplifier itself. Referring again to Figure 1, it can be seen readily that internal d-c paths are present, for example, *A B C D* in the power windings. Power consumed by circulating currents in these paths is derived in large part directly from mutual coupling with the control source. This power, of course, is not available for actual flux control within the reactors. Experience has definitely indicated that minimizing all induced direct currents in these paths, or elimination of them entirely, leads to a more efficient system.

Despite the aforementioned difficulties encountered when using the conventional magnetic amplifier circuitry, it should be pointed out that through careful design and engineering, servos can be built using such amplifiers which perform very satisfactorily within their design limits.[5]

The Reduction of Magnetic Amplifier Time Constant

In many cases, servo requirements are such that no excess time delay can be al-

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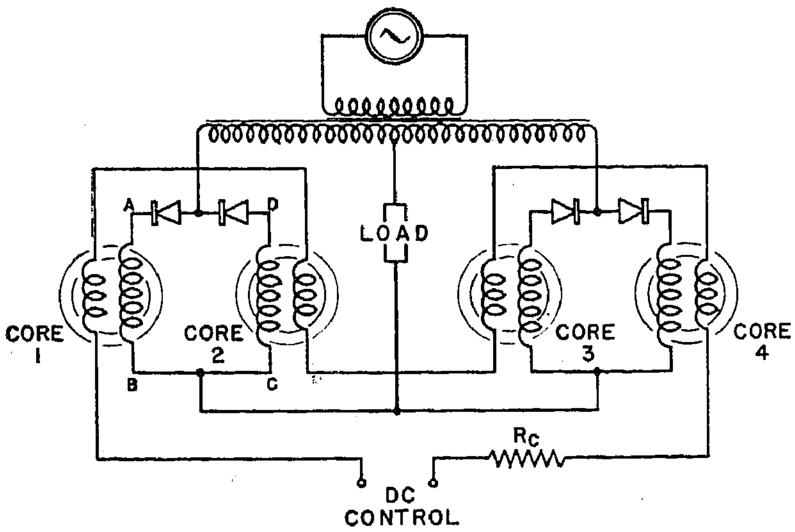

Figure 1 (left). A simple single-stage magnetic servo amplifier

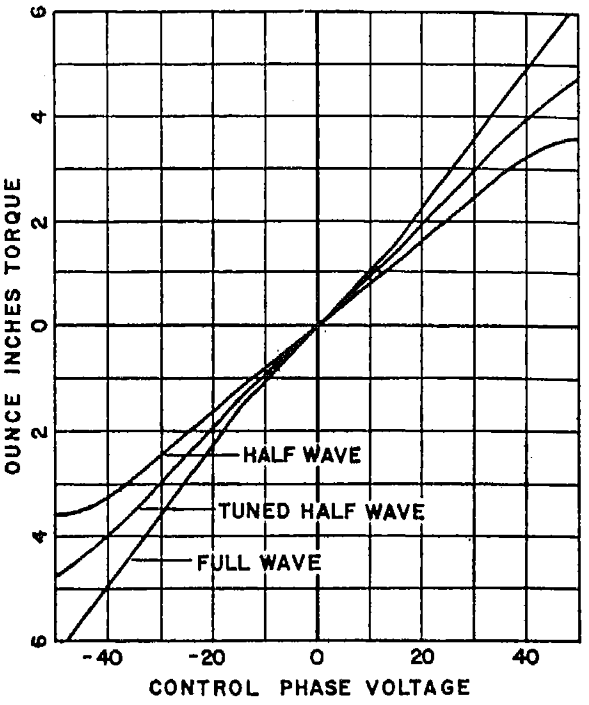

Figure 3 (right). Torque-voltage characteristics of 2-phase induction servo motor

lowed within the amplifier itself. One of the major problems in magnetic servo amplifier design and development, therefore, has been that of reducing the time constant, ideally, and if possible, to the absolute minimum of 1 cycle per stage. This must be done, of course, without undue sacrifice in gain.

The approach to this problem was to reduce in any way feasible the control winding L/R time constant. As pointed out previously, Z_L in any given control winding drops to a very low value when the reactor upon which it is wound goes into saturation. As long as conventional full-wave circuits are used, separation of the control windings such that only saturated, or unsaturated, cores are involved is difficult. If, on the other hand, only half-wave output is desired, the objective is achieved readily. Referring to Figure 1, if cores number 2 and number 4 are removed, the output is half-wave, but still with phase reversibility. Such a circuit is shown in Figure 2. In normal operation both cores will "fire," or saturate, during the operating half-cycle (that is, the half-cycle when voltage appears across the load). It can be seen readily that during this period the Z_L in both control windings is reduced to a very low value. Thus L/R drops to a low value and a definite control current is established. Actually, in the interval between the firing of one core and the other, the control winding on the unfired core acts as a voltage source due to mutual coupling with the power circuit. The voltage induced in the control circuit is always in such a direction as to aid control current build-up.

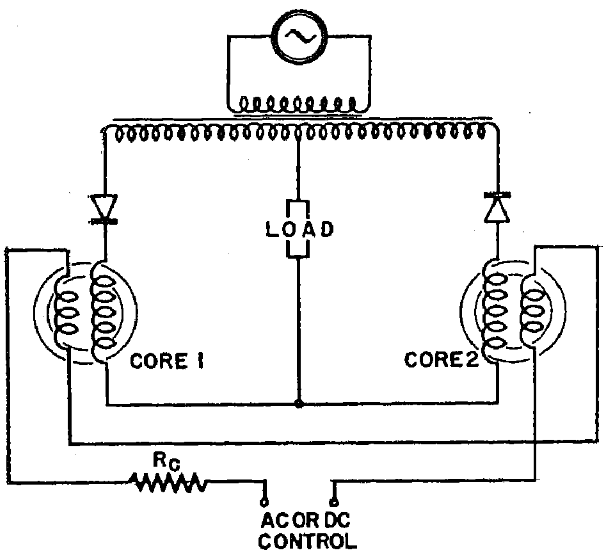

Figure 2. Half-wave magnetic servo amplifier with center-tapped power transformer

Since during the next operating half-cycle no output is obtained, this half-cycle is reserved for the purpose of controlling or presetting of flux. Thus the control current flowing at the end of the operating half-cycle is the current available at the beginning of the nonoperating or flux setting half-cycle. At any given level of operation, this same current is always established when both cores fire. If the same current is available at the beginning of each flux setting half-cycle, it is apparent that whatever flux level this current is capable of establishing during this period, will be established during each succeeding flux setting half-cycle. The operation is cyclical and, furthermore, no d-c transients in the control circuit are allowed to last longer than the period between the times when both cores are fired at the same time. It follows that the response of the system to a given control signal cannot exceed 1 cycle.

Following this same reasoning, it is apparent that a properly phased a-c control signal, or a properly phased pulse control signal will be effective in the half-wave circuit. It need only be established that at any given signal level the same voltage time integral is applied to the control winding during the nonoperating or flux setting half-cycle. If this is true, a cyclic operation is obtained in which no transients last longer than 1 cycle, thus assuring 1-cycle speed of response.

The fact that either direct current, alternating current or pulses, or for that matter combinations of these, are usable as control signals, makes the half-wave circuit quite flexible from the standpoint of control source requirements. Since at zero signal, no voltage appears across the output or load, the circuit is also compatible to cascading one stage on another. One drawback could be that the half-wave output would not be suitable for driving a 2-phase a-c induction motor. Although this output is unidirectional for any given phase sense input, the wave form which is applied to the motor will contain a high fundamental a-c component. Tests made on a typical 2-phase induction motor prove, however, that the torque obtainable with a half-wave signal is at least half that obtained with full-wave, and somewhat greater if capacitors are used across the motor to tune it to the fundamental frequency. Figure 3 shows the torque versus voltage curves of a 2-phase induction servo motor (5-watt low-inertia servo motor, Ford Instrument Company) for half-wave, tuned half-wave, and full-wave voltage.

The half-wave circuit fulfills the basic requirements for a servo amplifier, and in addition possesses the minimum speed of response possible for a magnetic amplifier. For these reasons this investigation of half-wave circuitry for use in high performance instrument-type servos has been carried out. The circuits developed and servo performance obtained using them are herein described.

Description of the Half-Wave Magnetic Servo Amplifier

The Bridge Circuit

The basic half-wave circuit shown in Figure 2 was obtained, as previously mentioned, by removing two reactors from the conventional full-wave parallel self-saturating circuit of Figure 1. This

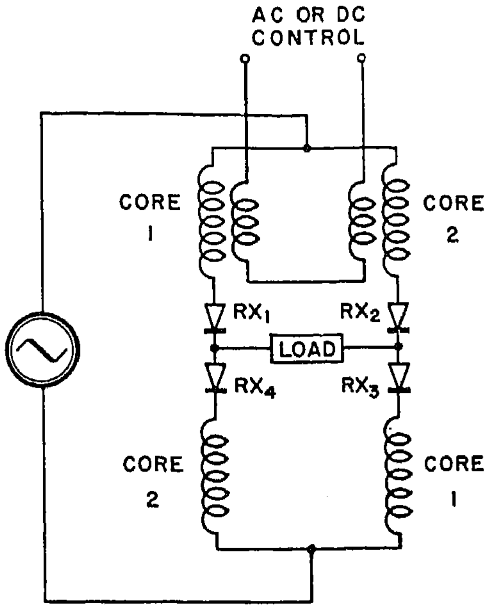

Figure 4. The basic half-wave magnetic servo amplifier bridge circuit

circuit possesses the advantages mentioned for use as a servo amplifier and, therefore, may be used as such. However, in actual design considerations it is desirable to keep components to an absolute minimum. One saving in this respect is to eliminate the center tapped power transformer by going to a bridge arrangement. Such a bridge circuit is shown in Figure 4. This circuit represents the basic design used in developing half-wave servo amplifiers. As indicated in Figure 4, opposite legs of the bridge are wound on the same reactor cores. Hence, only two cores are necessary although four could be used. It is preferable to wind the opposite bridge legs on the same core since for balanced operation they must act together. In this way, uniform operation is insured. For half-wave operation only two rectifiers, either RX_1 and RX_2, or RX_3 and RX_4 are absolutely necessary. In this circuit, however, all four rectifiers are used to completely eliminate any possibility for circulating currents in any leg or section of the bridge. It is to be noted that the load circuit, which may be either the control winding of the succeeding stage or the motor, is completely isolated from the bridge proper in that voltages induced in the load circuit cannot produce currents in the bridge.

Control of the Bridge Circuit

In the half-wave circuit the action of the control can be considered in a much simpler manner than in full-wave circuits. This is true since a complete half-cycle, the flux setting half-cycle, is available for the sole purpose of establishing control. During this period, the power winding circuit is inactive and its effect upon the control may be neglected. In full-wave circuits this, of course, cannot be done since some of the control windings are on reactors which are in their operating half-cycle.

Fundamentally, the requirement of the control is to establish a certain desired flux level in the reactor core during the flux setting half-cycle. This flux level essentially determines the firing angle of the reactor on its operating half-cycle.[6] Figure 5(A) shows a hysteresis loop which may be assumed to represent either reactor in the bridge because both are cycling around their respective loops in the same way at the same time if the bridge is in balance (that is, zero output). At the end of the operating half-cycle the reactor flux returns to residual point A. The control is then required to set a flux level, depending upon output desired somewhere between, say point D and C during the next half-cycle. It is not necessary that all the core losses involved in setting this flux level be supplied by the control source. In Figure 5(B) the basic bridge circuit is shown with the addition of another set of windings. These windings, called the reference windings serve the purpose of presetting a definite flux level, say B, in each reactor during the flux setting half-cycle. The power required to do this is obtained from the main a-c power source. The control source must then supply only the incremental power required to override the reference winding and shift the flux either up or down from point B. If the core material is quite rectangular, for example Orthonol, this incremental power may be very small. The fact that the control source is no longer required to overcome the coercive force of the core material opens the way for use of core materials having relatively large coercive force even in low level stages. The main requirement is that the material possesses a high incremental a-c permeability.

With reference to Figure 5(B) the action of the reference winding is as follows:

The main bridge power windings N_1, N_2, N_3, N_4 are active only when the a-c line polarity is positive at point X. (Determined by rectifiers RX_1, RX_2, RX_3, RX_4.) At the beginning of the operating half-cycle, that is, when point X goes positive, the core flux is at, say point B, Figure 5(A). The flux is then carried up the hysteresis loop to saturation where it remains until the end of this half-cycle, at which time the flux returns to residual, point A. During this period, no current flows in the reference windings N_{r1}, N_{r2} because of rectifiers RX_5, RX_6. The next half-cycle, however, line polarity is reversed and no current flows in windings N_1, N_2, N_3, N_4 but does in windings N_{r1}, N_{r2}. The currents flowing through N_{r1} and N_{r2} set up a magnetomotive force on the reactors opposite to that in the main power windings. Hence, the flux change brought about in the reactor is opposite. Thus, with zero control winding current the flux can be made to move to a predetermined level by the action of the reference windings. The required flux reference level, say point B, is set independently on each reactor by proper adjustment of the number of turns on the reference windings and the series resistance R_1 and R_2. In practice, rectifiers RX_5 and RX_6 may be eliminated if N_{r1}, N_{r2} and R_1, R_2 are properly determined. However, control of exact reference flux level is more critical without them.

The fact that each reactor may be independently adjusted or preset to the same reference flux level, regardless of the core's magnetic properties, is of considerable advantage in obtaining balanced operation from cores that normally are too unlike, magnetically, to be used in the same circuit. The "core matching" problem of course, is not completely eliminated. If two cores are dissimilar, although the same reference flux level may be readily established, the incremental flux change brought about by the control winding itself may be quite different. The important thing is that the incremental a-c permeability of the two cores

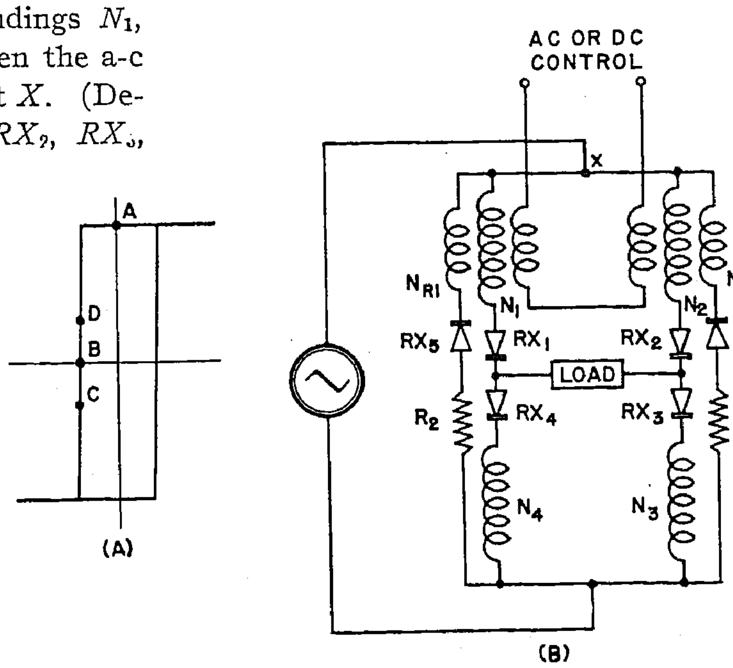

Figure 5(A). Hysteresis loop of reactor cores. (B). The half-wave bridge circuit with reference winding

be approximately matched in a region about the reference flux level so that the control has an equal effect on each reactor. The coercive force of the cores may be dissimilar and the actual shape of the a-c hysteresis loop near the saturation level may be dissimilar and balanced operation still obtained by proper adjustment of the reference windings

It would appear from the foregoing discussion that a separate reference winding is not necessary. A controlled back magnetomotive force, as furnished by the reference winding, could be obtained by shunting rectifiers RX_1, RX_2, RX_3, RX_4 and allowing a reverse current to flow through the power windings N_1, N_2, N_3, N_4. (A certain amount of this current does flow due to rectifier back leakage in actual practice. In the discussion, however, this has been neglected to simplify the argument.) The circuit would then be as shown in Figure 6. Actual measurement on these two circuits, however, shows a reduction in gain when shunted rectifiers are used. This is believed to be due to circulating currents which can flow around the bridge through these shunts. The normal turns ratio between the control windings and the power windings is such that the reflected impedance of these shunts is such as to make them appear as a noticeable load to the control circuit. On the other hand, the turns ratio between the control and reference windings can be made such that the impedance in the reference circuit is reflected as a high impedance into the control circuit. It is pointed out that back leakage in the power circuit rectifiers, whether due to deliberate shunting or through inefficiency in the rectifier, cause a flux change in the desired direction, that is, down the back side of the hysteresis loop. It is not the serious matter it can be in the conventional full-wave self-saturating circuit where the degree of positive feedback is determined by rectifier leakage. Frequently, ambient temperature variations, required in a given specification, may require a certain amount of shunting of rectifiers to minimize their temperature characteristics to the point where they can be used. In this case the degradation in amplifier gain must be tolerated to obtain the increased stability, and the circuit shown in Figure 6 would be used instead of that in Figure 5(B).

In the control circuit of the half-wave magnetic amplifier, the L/R time constant is of no great importance since d-c transients cannot exist over a complete operating cycle.[6] There is, therefore, no necessity for placing resistance or other passive elements in this circuit other than as might be demanded by the control source itself. Where synchro components are used for error detection in the servo system, it is frequently desirable to keep a minimum impedance of 10,000 ohms in the input circuit. It is conventional

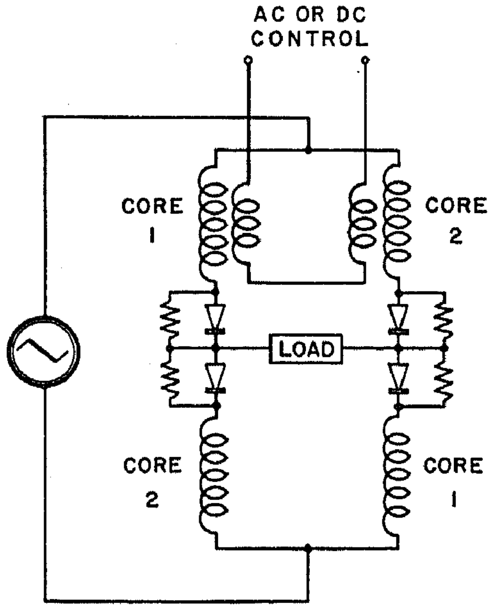

Figure 6 (left). The half-wave bridge circuit with shunted rectifiers

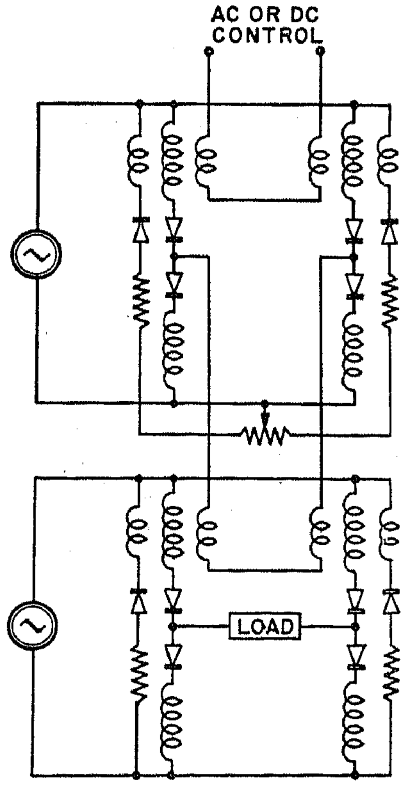

Figure 7 (left). Basic 2-stage half-wave magnetic servo amplifier using the bridge circuit

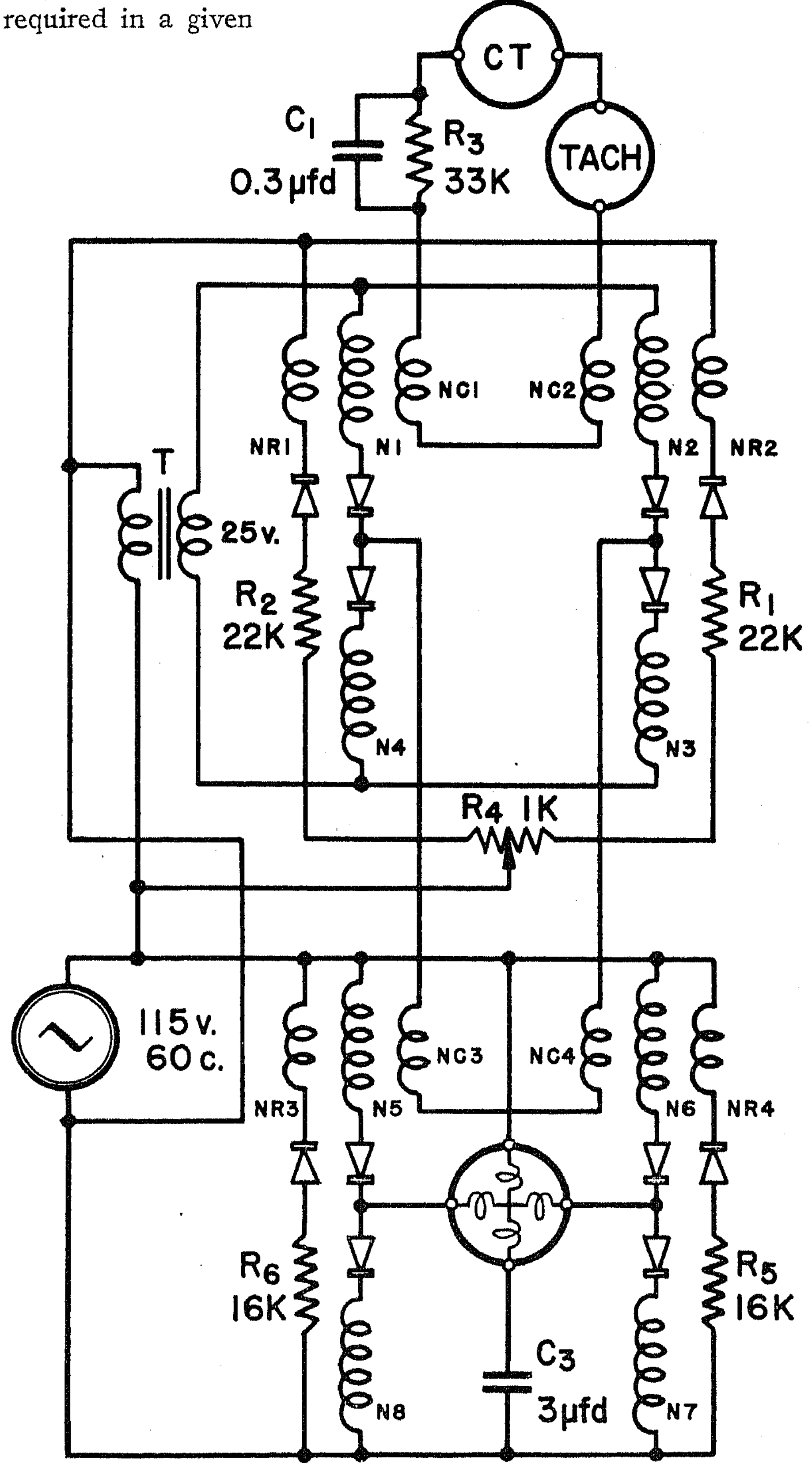

Figure 8 (right). Circuit diagram of 60-cycle half-wave magnetic servo amplifier

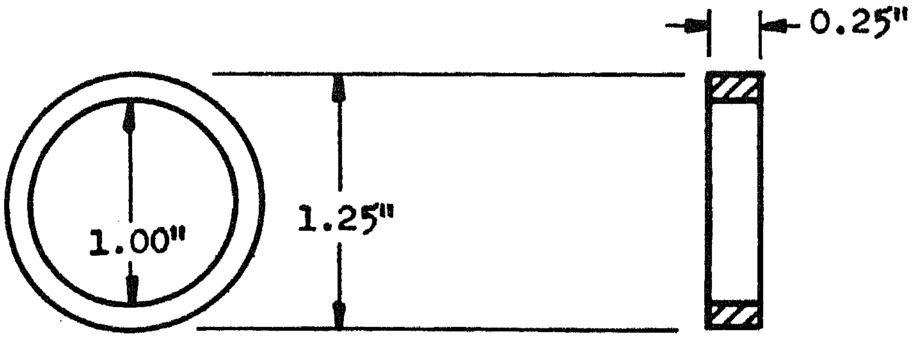

Figure 9. Core and winding data for 60-cycle amplifier input stage

Details of Magnetic Cores

Material	Orthonol (50-per-cent Ni 50-per-cent Fe alloy, grain oriented, dry hydrogen anneal)
Configuration	Tape wound toroid
Inside diameter	1.00 inch
Outside diameter	1.25 inches
Tape, width	0.25 inch
Tape, thickness	0.002 inch
Insulation	MgO applied in winding process

Details of Windings

Windings	Function	Number Turns	Wire Size
N_1 N_2 $N_3 N_1$	Power	2,000	Number 33
N_{R1} N_{R2}	Reference	500	Number 36
N_{C1} N_{C2}	Control	100	Number 36

Each core has two power windings, a reference, and a control winding

practice to insure this minimum by insertion of a resistance in the input control circuit. The disadvantage of doing this is loss of gain and limiting of current in the control winding, as has been previously discussed. A large number of turns on the control windings is usually required to obtain a reasonable gain because ampere turns of control becomes the determining factor.

INCREASED GAIN THROUGH REDUCTION IN CONTROL TURNS

To set down equations for flux level in terms of ampere turns magnetomotive force on a reactor having a highly rectangular hysteresis loop is virtually impossible, since the ampere turn, or H, and flux level B relationship is nearly discontinuous in many magnetic materials available today. This difficulty has been pointed out clearly and a more satisfactory analysis of magnetic amplifier performance in terms of voltage time integrals has been made by R. A. Ramey.[7] In order to make use of this method, however, it is necessary that the voltage time integral which appears on the windings of the reactors be known. Normally, this information is as difficult to obtain precisely as is the flux level in terms of ampere turns. The instantaneous currents flowing in a circuit involving resistance and reactance due to a magnetic amplifier reactor are so complex that although the voltage time integral applied to the system externally may be known exactly, the portion of this which finally appears across the reactor winding itself, and which uniquely determines $d\phi/dt$ in the core, is difficult to establish. The basic approach, or method of analysis is, nevertheless, extremely useful in terms of understanding what actually takes place in the magnetic amplifier.

Ideally, if a control circuit contained no resistance and the control voltage time integral were known, the amount of flux change which would be obtained by this control is given by Faraday's law as

$$\Delta\phi=\frac{1}{N}\int E_c dt$$

where

N = Number of turns of control winding
$\int E_c dt$ = Voltage time integral over the flux setting half-cycle (for the half-wave circuit)

For a given level of control, that is, $\int E_c dt$ fixed, the maximum flux change would be obtained by making N as small as possible. In other words, if voltage control is utilized, the voltage gain of an amplifier is increased if the number of control turns is decreased. To insure voltage control, it is necessary that the control be a low impedance source. Of course, in practice it is possible to get so low in control turns that the current drain on the control source necessary to maintain a given voltage across the control windings may become excessive. There will be, consequently, an optimum number of turns on the control winding representing a compromise between the voltage and the current limitations of a given source.

The utilization of voltage control in the input control circuit, where resistance is usually present due to control source limitations, is not possible. However, the 2-stage bridge circuit lends itself to this principle very nicely. The output of the first stage may be fed directly into the control winding of the second stage with no passive element required. Thus, one may be assured that whatever voltage appears at the output of the first stage is impressed directly into the control winding of the second.

Conclusion

A 2-stage magnetic servo amplifier has been developed utilizing the principles set forth in the foregoing discussion. The basic circuit is shown in Figure 7. This circuit has been used in both 60- and 400-cycle applications to obtain performance much superior to that obtainable with conventional full-wave circuitry. The over-all maximum response time of this circuit is $1^1/_2$ cycles of the power frequency (1 cycle for first stage, 1/2 cycle

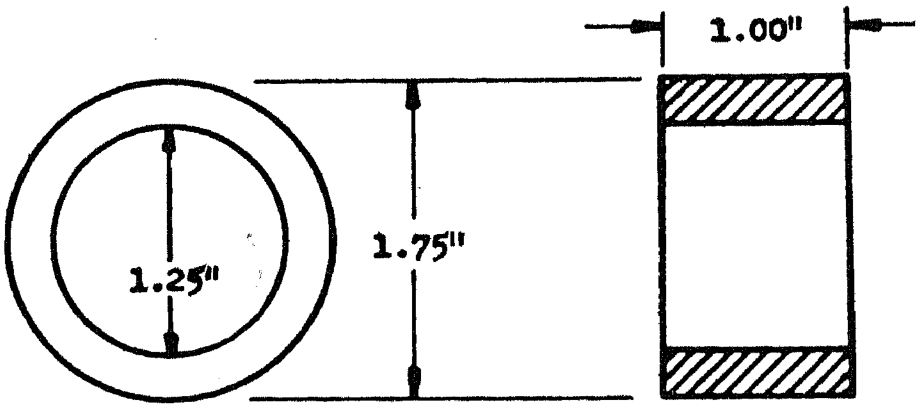

Figure 10. Core and winding data for 60-cycle amplifier output stage

Details of Magnetic Cores

Material	Orthonol (50-per-cent Ni 50-per-cent Fe alloy, grain oriented, dry hydrogen anneal)
Configuration	Tape wound toroid
Inside diameter	1.25 inches
Outside diameter	1.75 inches
Tape, width	1.00 inch
Tape, thickness	0.002 inch
Insulation	MgO applied in winding process

Details of Windings

Windings	Function	Number Turns	Wire Size
N_5 N_6 N_7 N_8	Power	1,500	Number 30
N_{R3} N_{R4}	Reference	500	Number 34
N_{C3} N_{C4}	Control	25	Number 28

Each core has two power windings, a reference, and a control winding for each additional stage). The ease with which additional stages may be cascaded is, of course, evident. In comparison to conventional full-wave circuitry only about one-half as many components, occupying approximately one-half the volume, are required.

The complete design and adjustment details for typical 60- and 400-cycle half-wave servo amplifiers are shown in Appendix I, and the performance characteristics of the two complete servo systems are shown in Appendix II.

This report has been written purposely in a general nature to introduce the basic concepts underlying the development of the amplifier circuitry described. The utilization of half-wave magnetic amplifiers for instrument servos represents a new approach to this problem. The fact that delay time in the amplifier has been reduced to the absolute minimum opens the way for further development and utilization of magnetic amplifiers in high performance servo systems heretofore requiring vacuum tube amplifiers.

Appendix I. Design and Adjustment Details for Typical 60- and 400-Cycle Half-Wave Servo Amplifier

60-Cycle Half-Wave Servo Amplifier Circuit

The complete circuit diagram of a 60-cycle servo amplifier based on the half-wave bridge is shown in Figure 8. This circuit will be recognized as basically that of Figure 7. All cores are of a 50-per-cent iron 50-per-cent nickel grain oriented material having a very rectangular hysteresis loop. The exact core and winding data for the input stage are given in Figure 9 and for the output stage in Figure 10.

Each of the four output stage power windings is wound to absorb about 75 per cent of full-line voltage. To permit smaller cores, the input stage is operated at 25 volts obtained from transformer *T*. Since the current demands made upon the input stage are rather heavy, this transformer must be capable of delivering approximately 125 milliamperes continuously. Each input stage power winding is wound to absorb the full 25 volts.

The reference windings of both stages are 500 turns each. Since final adjustments are made in the series resistors in the reference circuits, this number is not critical. It should be high enough that excessive currents are not required for referencing, but not so high that valuable winding area is sacrificed. The mutual coupling between the reference winding and control circuit becomes critical also if reference turns are too great.

The input stage control windings are 100 turns each; however, a higher gain would result in the amplifier if these turns were increased. It is recognized that when output is desired, the two reactors do not fire together and during the interval when one is fired and the other is not, voltage is induced in the control circuit. Therefore, these windings should be kept to a minimum to reduce this value of induced voltage.

The output stage control windings are very low in keeping with the voltage control concept. The values used (25 turns) were experimentally arrived at as being the best compromise for the input stage shown. However, the operation of the amplifier is not noticeably changed if this number is reduced to 8 or 10 turns.

All fixed resistors shown are 1 watt carbon, each experimentally determined for the particular core and its desired operating conditions. R_3 is a 4,000-ohm potentiometer used to bring the amplifier into balance.

The network R_3C_1 in the input circuit is used primarily to guarantee a minimum of 10,000 ohms input impedance. The *RC* combination allows a slight phase adjustment found to be helpful in obtaining optimum operation, which a simple resistance would not permit.

The rectifiers in the power circuit of the input stage consist of four $^5/_8$-inch-square selenium plates, doubler connected (Radio Receptor Company type *J* plates in intimate contact). The input and output stage reference winding rectifiers consist of eight $^1/_2$-inch-square selenium plates, half-wave connected (Radio Receptor Company type *Y* plates in intimate contact).

The output stage power circuit rectifiers consist of 18 1-inch-square selenium plates, doubler connected (Radio Receptor Company type *18M0488*).

This amplifier was designed to operate a Ford Instrument Company 5-watt low-inertia servomotor. The fixed or excitation phase of this motor is wound to operate in series with a 3.5-microfarad capacitor directly from a 115-volt 60-cycle line. The control phase has two windings which can be operated in series at 115 volts or in parallel at 57.5 volts. In this design, the control windings are used in parallel presenting an impedance of about 250 ohms to the amplifier output.

The amplifier is adjusted to operate about a quiescent firing angle (that is, point during operating half-cycle where cores saturated with zero signal input) of 90 degrees in the input stage and approximately 150 degrees in the output stage. These quiescent firing angles are set individually on each core by adjusting the series resistors in the reference windings. It is preferable to operate about a 90 degree

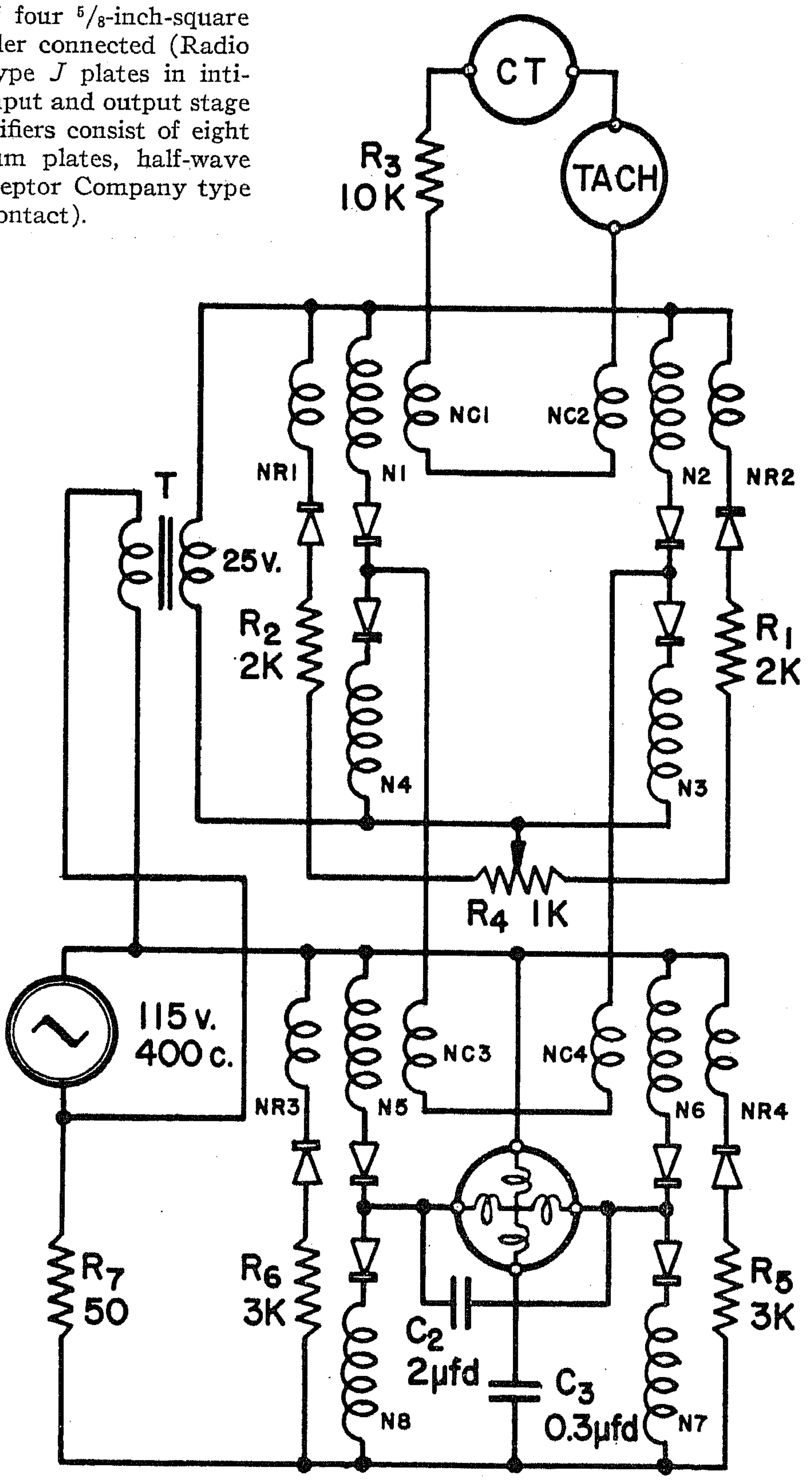

Figure 11. Circuit diagram of 400-cycle magnetic servo amplifier

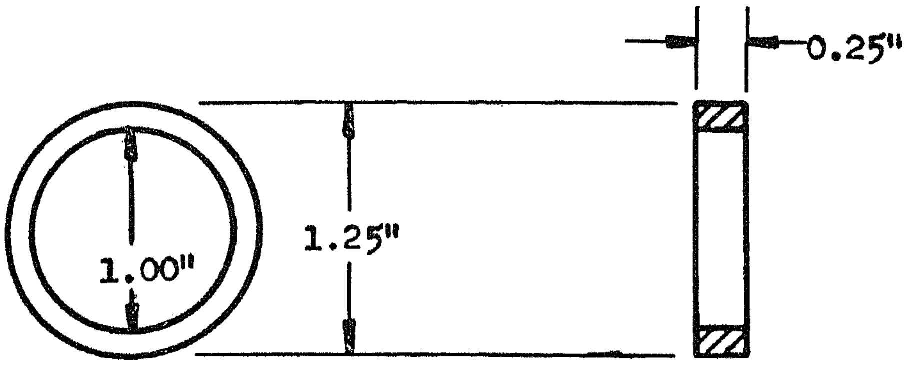

Figure 12. Core and winding data for 400-cycle amplifier input stage

Details of Magnetic Cores

Material	Orthonol (50-per-cent Ni 50-per-cent Fe alloy, grain oriented, dry hydrogen anneal)
Configuration	Tape wound toroid
Inside diameter	1.00 inch
Outside diameter	1.25 inches
Tape, width	0.25 inch
Tape, thickness	0.002 inch
Insulation	MgO applied in winding process

Details of Windings

Windings	Function	Number Turns	Wire Size
$N_1\ N_2\ N_3\ N_4$	Power	500	Number 28
$N_{R1}\ N_{R2}$	Reference	50	Number 31
$N_{C1}\ N_{C2}$	Control	300	Number 36

Each core has two power windings, a reference, and a control winding

firing angle in each stage, since at this point maximum output is obtained for a given differential in the firing angle of each core. It is not desirable to operate the output stage of this amplifier about 90 degrees, however, because the quiescent currents flowing around the bridge are excessively high causing heating of the rectifiers and power winding. For this reason, the output stage quiescent current is reduced to a lower value by firing the cores at about 150 degrees. The over-all efficiency of any given stage can be increased by operating the stage closer to cutoff value (that is, firing angle of 180 degrees). If the stage is never to be operated at cutoff, it is not necessary that each power winding absorb full-line voltage but only the voltage time integral necessary to maintain the desired firing angle. The maximum output cannot be obtained, however, unless each winding will absorb full-line voltage. In this connection, it should be pointed out that the operation of a given stage is never symmetrical. If the firing angle of one core is advanced, say 5 degrees, by the control action, the other core is not retarded 5 degrees. This is true because when the first core fires full-line voltage, less drop across series rectifier and winding of first core, appears across each winding of the second. On the other hand, before the first core fires line voltage appears across both windings of the second in series. Hence, when one core fires, $d\phi/dt$ in the other is practically doubled so that it fires sooner than it would normally if the line voltage always appeared in series with its power windings instead of suddenly in parallel with them.

The choice of core size and power winding turns is determined primarily on the basis of line voltage to be controlled and the load current requirements. For example, if full-line voltage is to be controlled, the core size and power winding turns must be such as to absorb approximately 115 volts on each winding.

This can be accomplished by any combination of core size and power winding turns which will satisfy the following well-known expression

$$E=\frac{4.44BNAf}{10^8}$$

where

E = voltage absorbed by reactor
B = maximum flux density
N = number of turns on power winding
A = core cross-sectional area
f = power frequency

However, in designing a reactor, the power winding size and the number of turns are held to values which will produce a low-voltage drop across the reactor during saturation.

The procedure is usually, first, to determine the power winding wire size necessary to carry the required load current without excessive heat losses, then, to use a core size which has a sufficient window area to allow winding the required number of turns of this wire size. The number of turns, of course, depends upon the core size, but normally only one or two test calculations are necessary before the right combination of core dimensions and winding turns are arrived at. Where standard core sizes are available, the nearest core size to meet these requirements would be chosen without concern over whether this particular core size to copper area ratio is the exact optimum.

400-Cycle Half-Wave Servo Amplifier Circuit

The complete circuit diagram of a half-wave 400-cycle amplifier is shown in Figure 11. Again the similarity to Figure 7, and Figure 8, is evident. The general discussion in the previous section relating to design and adjustment considerations of the 60-cycle amplifier apply also to this circuit, hence, will not be reiterated. The exact core and winding data for the 400-cycle input stage are shown in Figure 12, and for the output stage in Figure 13. Transformer T must deliver 25 volts at 250 milliamperes. All resistors, with the exception of R_p, are 1 watt carbon, each experimentally determined for the particular core and its desired operating conditions. R_p is a 50-ohm 10-watt wire wound resistor. No input circuit RC network is used in the 400-cycle amplifier since phase shift adjustments in this unit were not found necessary.

In this amplifier, both input and output stage are adjusted to a quiescent firing angle of 90 degrees, for maximum performance. As previously stated, adjustment of the output stage to this quiescent level allows rather heavy currents to flow around the bridge. To reduce this current to a satisfactorily low value, without deviating from the 90-degree firing point, as was done in the 60-cycle amplifier, the series resistor R_p is inserted in the main power circuit. Although voltage dropped in this resistor is not available for motor operation, the loss here is more than offset in increased gain by operating the output stage at a 90-degree firing angle. Of course, R_p

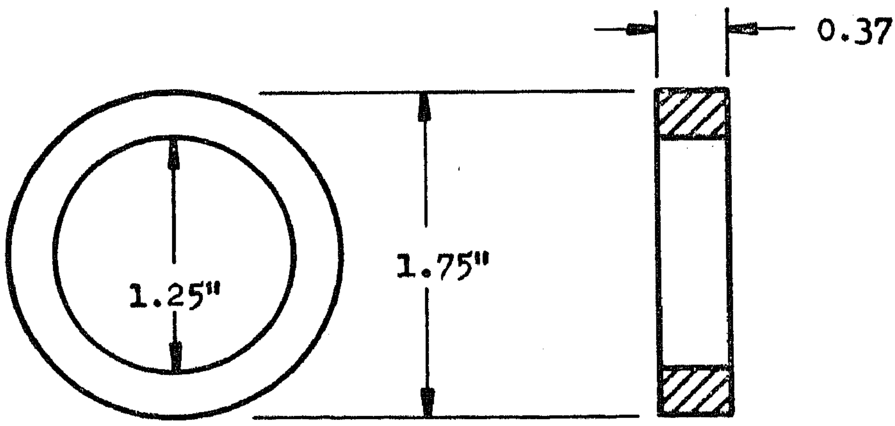

Figure 13. Core and winding data for 400-cycle amplifier output stage

Details of Magnetic Cores

Material	Orthonol (50-per-cent Ni 50-per-cent Fe alloy, grain oriented, dry hydrogen anneal)
Configuration	Tape wound toroid
Inside diameter	1.25 inches
Outside diameter	1.75 inches
Tape, width	0.375 inch
Tape, thickness	0.002 inch
Insulation	MgO applied in winding process

Details of Windings

Windings	Function	Number Turns	Wire Size
$N_5\ N_6\ N_7\ N_8$	Power	850	Number 29
$N_{R3}\ N_{R4}$	Reference	50	Number 31
$N_{C3}\ N_{C4}$	Control	10	Number 22

Each core has two power windings, a reference, and a control winding

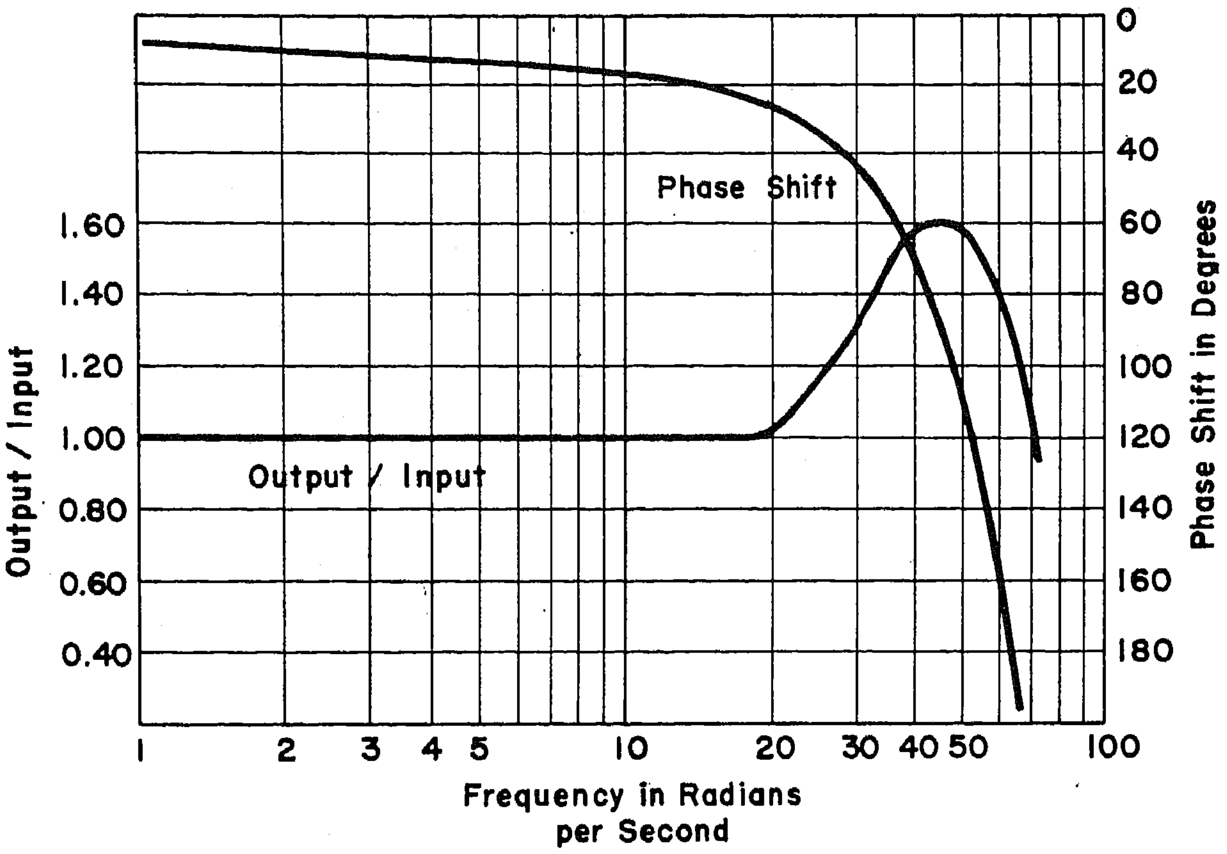

Figure 14. Closed-loop system response (60 cycle)

must be less than a certain maximum resistance if this is to continue to be true, for eventually the voltage drop across it would cause a decrease in over-all gain. The value for R_p shown, 50 ohms, was chosen to give maximum voltage gain of the amplifier.

The rectifiers in the power circuit of the input stage consist of two, 1-inch-square selenium plates, doubler connected (Radio Receptor Company type *2MO454*), and the input stage reference winding rectifiers are *1N34* germanium diodes.

The output stage power rectifiers consist of 12, 1-inch-square selenium plates, doubler connected (Radio Receptor Company type *12MO447*), and the output stage reference winding rectifiers are the same as in the 60-cycle amplifier reference circuits.

This amplifier was designed to operate a *Mk 16 Mod O*[8] low-inertia servo motor. The fixed phase of the Mk 16 motor operates in series with an 0.3 microfarad capacitor directly from a 115-volt 400-cycle line. The control phase has two windings which can be operated in series on 115 volts or in parallel on 5705 volts. In this design, the control phase is parallel connected and presents an impedance of 365 ohms to the amplifier output.

Appendix II. Performance Characteristics of Instrument Servos Using 60-Cycle and 400-Cycle Half-Wave Amplifiers

The 60-cycle amplifier described was evaluated in a closed-loop instrument-type servo of the remote positioning type. The gear reduction from the motor to the synchro current transformer shaft was 22.5-to-1 and the system was stabilized with an a-c tachometer generator (induction generator type *945B-01600*, Kollsman Instrument Corporation) geared directly to the motor. The tach output was fed back directly in series with the current transformer error signal.

The 400-cycle amplifier was evaluated in a similar closed loop servo. The gear reduction from the motor to the synchro current transformer shaft was 30-to-1. The system was stabilized with an a-c tachometer generator built directly on the motor shaft. This tachometer output was also fed directly in series with the current transformer error signal. (Advantage may be taken of the internal impedance of the tach so that if 10,000-ohm impedance is necessary in the current transformer rotor circuit the additional circuit impedance may be only that required to equal 10,000 ohms when added to the tach impedance.)

The performance characteristics given in Tables I and II for each servo are based on the following quantities

Maximum Slewing Rate. This is the highest angular velocity which the servo system will continuously follow when energized by a synchro generator whose rotor is continuously rotated.

Following Rate. The following rate is the angular velocity at which the velocity error reaches one degree.

Static Error. The static error is the maximum difference in angular position between the current transformer in the servo system and the synchro generator which excites it when the system is at rest. This figure does not include drift error.

Frequency Response. The frequency response data was obtained using servo frequency analysis equipment similar to that described in *NOLM 10930.*[9] A suppressed carrier signal of variable amplitude and frequency is applied to the system. The amplitude and phase of the output is then measured. From this data, system stability and performance are obtained by conventional methods.

Voltage Gain. This is the voltage gain of the magnetic amplifier with the current transformer utilized as a signal source and the servo motor (stalled torque conditions) as a load on the amplifier. Both motor and current-transformer voltages were read on a Ballantine model *300 VTVM.* This must be taken into consideration since the output wave form is nonsinusoidal and the indicated motor voltage depends upon the type meter used. The value shown is for small signal inputs within the linear range of operation of the amplifier.

All were taken at room temperature with line voltage of 115 volts 60 cycle (or 400 cycle). The gear trains were carefully adjusted to minimize back lash and sticking. Performance of the systems under extremes of voltage, frequency, and temperature as are required in military specifications are not herein given. The major difficulty encountered in meeting such specifications is almost always associated with drift in a

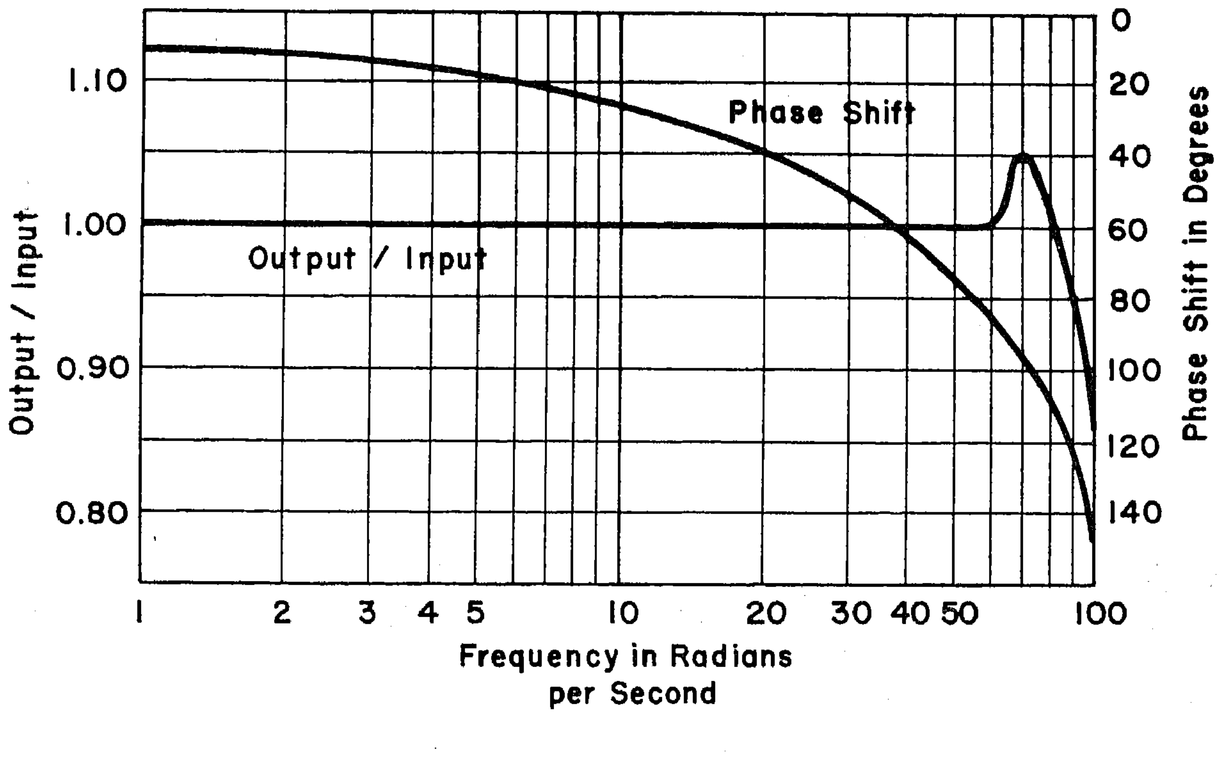

Figure 15. Closed-loop system response (400 cycle)

magnetic amplifier system. The performance of the half-wave amplifier in this respect will be comparable with previously reported tests for full-wave systems[5] because both are balanced bridge amplifiers and the causes of drift (bridge unbalance) are common to both. The elimination of drift is closely associated with core and rectifier matching. Of the two, the rectifiers are much more difficult to control and maintain. The data in Tables I and II are intended to give an idea of the dynamic servo performance characteristics of the half-wave magnetic amplifiers and should not be taken as the ultimate performance which might be obtained in a system completely engineered to meet rigorous military specifications.

Table I. Measured Performance of the 60-Cycle Servo

Maximum slewing rate	550 degrees per second
Following rate	1 degree per 60 degrees per second
Static error	less than 0.1 degree
Frequency response and phase shift	Figure 14
Voltage gain	12

Table II. Measured Performance of the 400-Cycle Servo

Maximum slewing rate	1000 degrees per second
Following rate	1 degree per 200 degrees per second
Static error	less than 0.01 degree
Frequency response and phase shift	Figure 15
Voltage gain	20

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No Discussion

Electronic Recorder with Range and Precision Adequate for the Platinum Resistance Thermometer

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MEMBER AIEE

THE platinum resistance thermometer is the interpolating device for the 1948 International Temperature Scale from −182.97 degrees centigrade to +630.5 degrees centigrade.[1–3] When properly made the reproducibility of its resistance justifies its calibration and use for measurements to 0.01 degrees centigrade or 1/80,000th of its temperature range.[3] Mueller attacked this difficult resistance-measurement problem with a hand-operated decade bridge.[4] Stull attacked the problem with a mechanically balanced recorder using a slide-wire plus decades[5] and explained the value of the recorder, one example being the determination of chemical purity by freezing point measurement.[6] While Stull was building his recorder the author and his associates were also building a recorder, using one of the several methods of electronic balancing which had then been developed.[7,8] Based on the experience with these two types of recorders there was developed a third type with improved precision, the experimental form of which is the subject of this paper. The commercial form may differ in details from the experimental form described here.

The recorder has a slide-wire with a span of 1.02 ohms. A 1-ohm decade and a 10-ohm decade are automatically switched to give a total range to the recorder of 0 to 100 ohms. Features will be described which permit unattended operation of the recorder with a precision approaching 0.001 ohm, which corresponds approximately to 0.01 degree centigrade for a 25-ohm platinum resistance thermometer.

Unattended Operation and Precision

In the following text are listed three items which are required for unattended operation that would not be required if an operator were in attendance to do certain things. There are listed also three items affecting precision which would exist even if an operator were in attendance. Taken together, these six items form a check list found useful to the author, which may assist the reader in understanding the paper.

Unattended operation

1. Automatic changing and recording of decade resistors.
2. Compensation for expansion of chart.
3. Smallness of zero offset of null detector.

Precision (possible sources of error)

4. Mechanical transmission from the slide-wire contact to mark on chart.

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The author wishes to give credit to D. R. Stull who, with his associates at the Dow Chemical Company, pointed out the need for an instrument of this general type,[6] and to acknowledge the many significant contributions of the late Frank F. Dern, Jr.

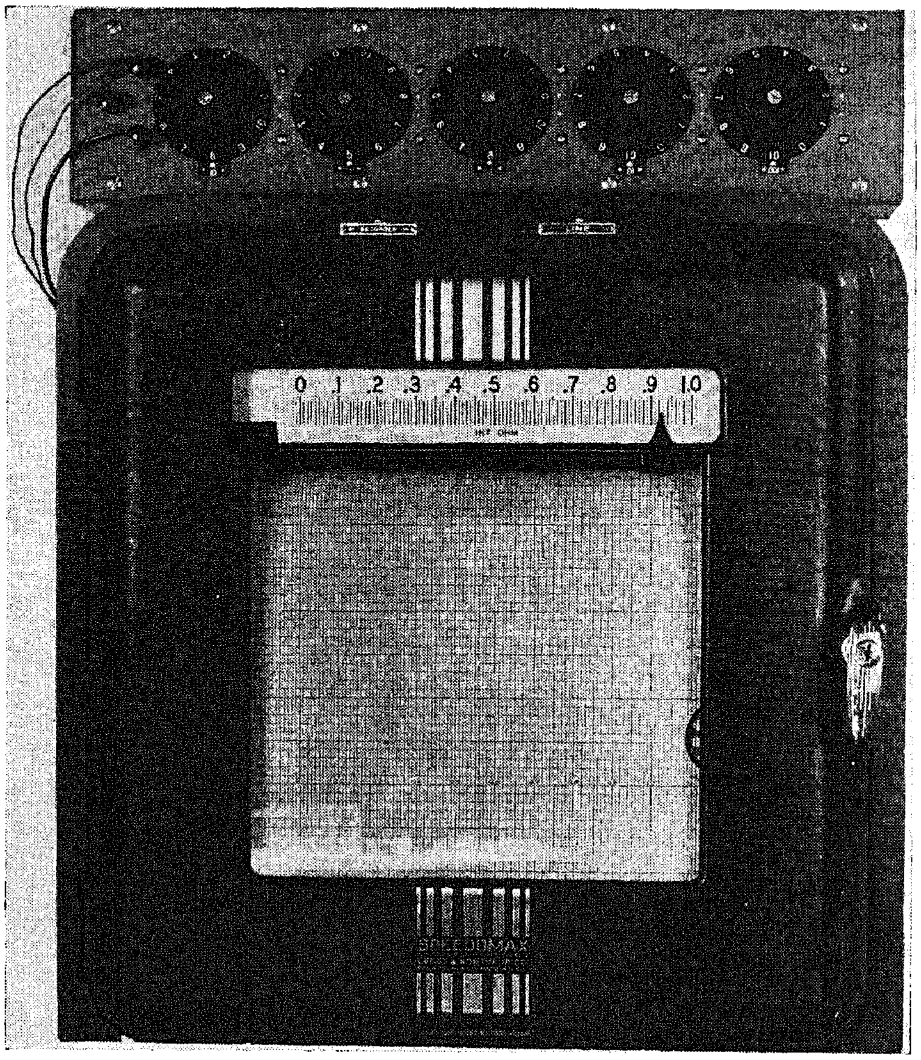

Figure 1 (left). Recorder and resistance box

Five figures may be read from the 5-dial box and five figures may be read from the recorder chart

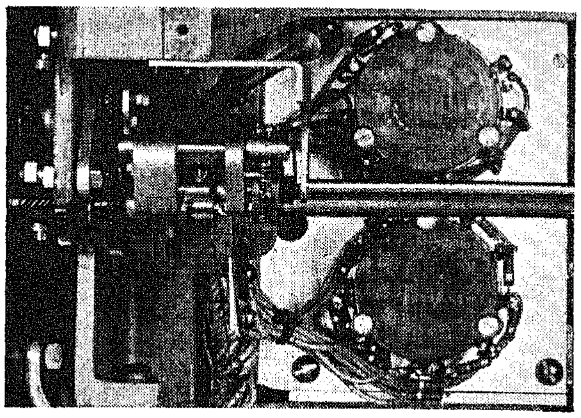

Figure 3 (right). Printer and decade switches

The interconnection is the diagonal shaft at the top

5. Setting of slide-wire contact on true balance position.

6. Changes in bridge zero, in slide-wire or in decade resistors between calibration and use.

Record of Changes in 5-Decade External Resistance Box

Figure 1 shows the recorder and a resistance box. The resistance box is not a part of the recorder but was used in place of the platinum resistance thermometer for the following tests. Five figures may be read from the 5-dial resistance box and five figures may be read from the recorder chart.

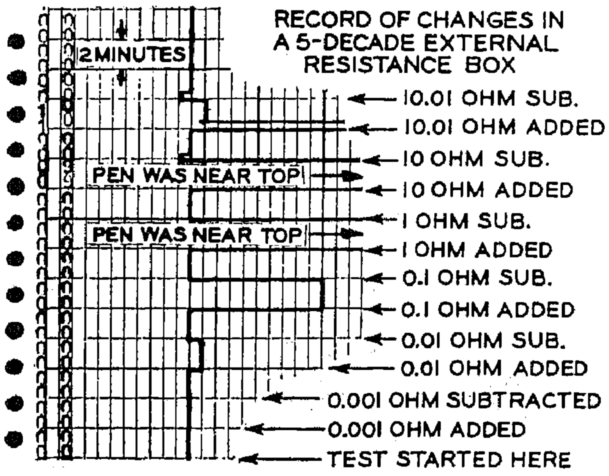

Figure 2. Record of changes in 5-decade external resistance box

Small values of resistance may be read from the pen line. Large values of resistance require reading of the printed figures also

The record of changes made in the 5-decade external resistance box, as shown in Figure 2, seems to require no explanation for the addition and subtraction of the 0.001-ohm, 0.01-ohm, and 0.1-ohm steps from the external resistance box. For the 1-ohm step, the pen moved 100 divisions and stopped to record at or very near to the 1.00-ohm chart line. For the 10-ohm step, the pen also stopped to record at or very near to the 1.00-ohm chart line, the other 9 ohms being recorded by the right-hand or units print wheel. Subtraction of the 10 ohms returned the print wheel and then the pen to zero. The print-wheel change was initiated by travel of the pen, and slide-wire shaft, to a position below zero, as shown on the chart. When 10.01 ohms were added, the pen moved to its uppermost position and remained there until the decades moved to 1 in the tens column and 0 in the units column, thus recording 10 ohms. The 0.01 ohm was recorded by the pen, coming in from the high side. Subtraction of the 10.01 ohms from the external box returned the print wheels and pen to zero.

Printer and Decade Switches

For Figure 3, the chart and chart roll have been pulled away to show the two print wheels in the center of the picture. The 1-ohm and 10-ohm decade switches are at the right. The mechanical interconnection between the switches and the print-wheel unit is the diagonal shaft at the top. The inking roller is just visible in its retracted position behind the print wheels.

A reversible capacitor-type motor drives the print wheels through gears and thence through a shaft shown at the top of Figure 4. The Geneva movement at the right drives the 10-ohm decade on the bottom shaft. The star wheel on the 1-ohm shaft operates an "index" switch which assists in stopping the motor. The trip switches, at the extreme lower left, start the motor in the proper direction whenever the pen approaches the upper or the lower limit of its travel.

The trip switches are shown from the rear in Figure 5. The arms on the slide-wire shaft are set to allow the pen to travel a little more than 1.00 ohm without tripping, to avoid frequent changes of the decades when the thermometer resistance is hovering around some integer value.

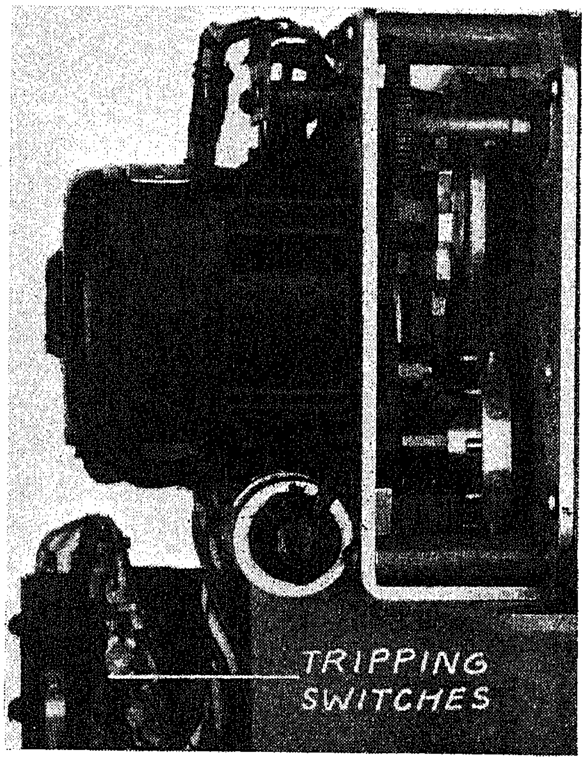

Figure 4. Drive for print wheels and decade switches

The gears drive the print wheels through shaft at top. The Geneva movement, at right, drives 10-ohm decade on bottom shaft. The star wheel on the 1-ohm shaft operates the index switch

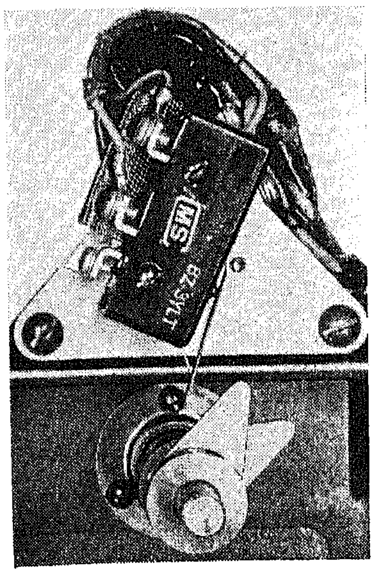

Figure 5 (left). Trip switches and actuating arms on slide-wire shaft

The arms are set to allow a slight amount of overtravel at both ends of slide-wire

Figure 6 (right). Decade-motor circuit

The index switch gives push-button action, braking, and precise positioning. The friction switch remembers the direction of motor start and so avoids the use of relays

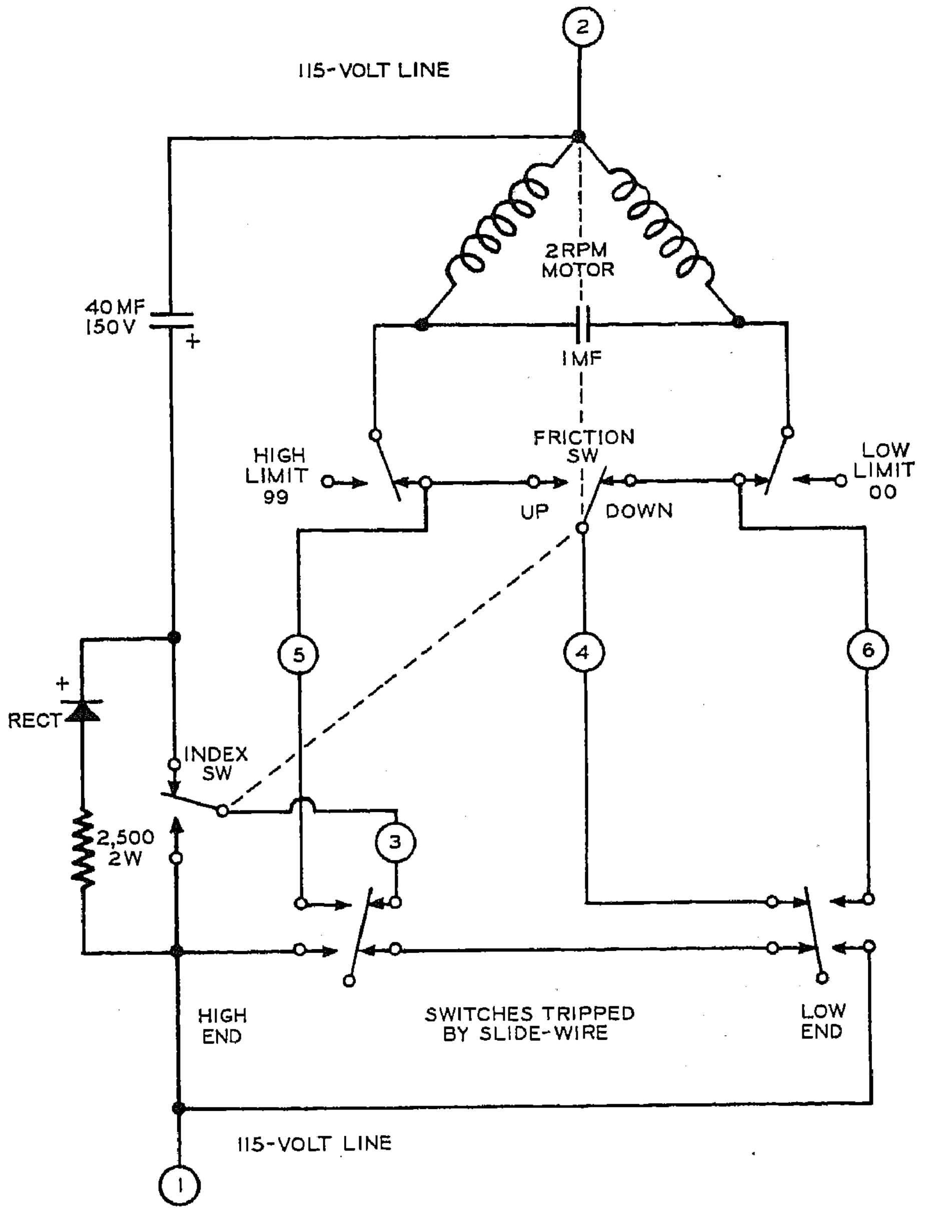

Figure 6 shows the decade-motor circuit. The index switch brakes the motor with direct current when the decade switches and print wheels have moved into the desired position. The "friction" switch reverses with reversal of the motor thus remembers the direction of motor start and so avoids the use of relays.

Humidity Compensator

Strip charts made of paper expand in width by several tenths of a per cent with increase of humidity and so could introduce errors of several tenths of a per cent. To avoid such errors, mechanical means as shown on Figure 7 have been used so that expansion of the chart moves the pen as well as the chart thus preventing relative motion from this cause.

Harrison[9] had this basic idea over 20 years ago. The execution of the idea for this recorder is different[10] and is most readily presented by Figure 8. The spider is positioned axially by the chart. Through the groove in its hub, via the engaging pin, a bell crank and a long lever running through the pen carriage to a pivot near to the other edge of the chart, the pen is moved in space with expansion of the chart so as to avoid motion relative to the chart. The long lever is pivoted near to the fixed edge of the chart so that its effect on the pen will be proportional to the pen's distance from the fixed edge of the chart, as it should be to avoid relative motion. The chart at right of Figure 8 shows the performance with increasing, then decreasing humidity. The line made by a fixed pen shows that the chart expanded about one-half of a division or 0.5 per cent, but the mark made by the compensated pen shows no change.

The Balancing Amplifier

The balancing amplifier as shown in Figure 9 has an extra stage of amplification which provides sufficient extra sensitivity to allow reduction of the thermometer current, from the 5 to 8 milliamperes which Stull[5] was forced to use, to 2 milliamperes as used by the National Bureau of Standards for calibration of precise platinum thermometers suitable for this recorder. Some of the spare gain can be used to make the motor responsive to random noise thus eliminating any persisting effect from friction and so obtaining extreme sensitivity useful for self-checking as will be explained later.

Any spurious voltage or noise which when referred to the input has a d-c equivalent or low-frequency equivalent is

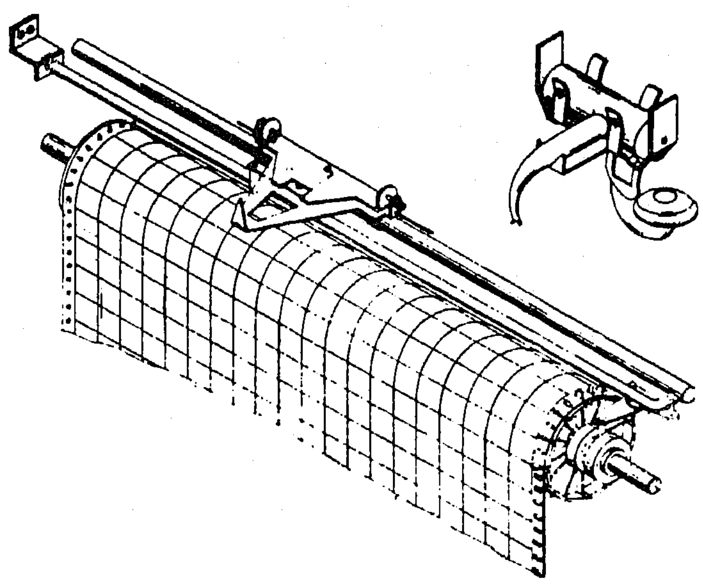

Figure 7 (left). Humidity compensator. Expansion of the chart moves the pen, thus preventing relative motion

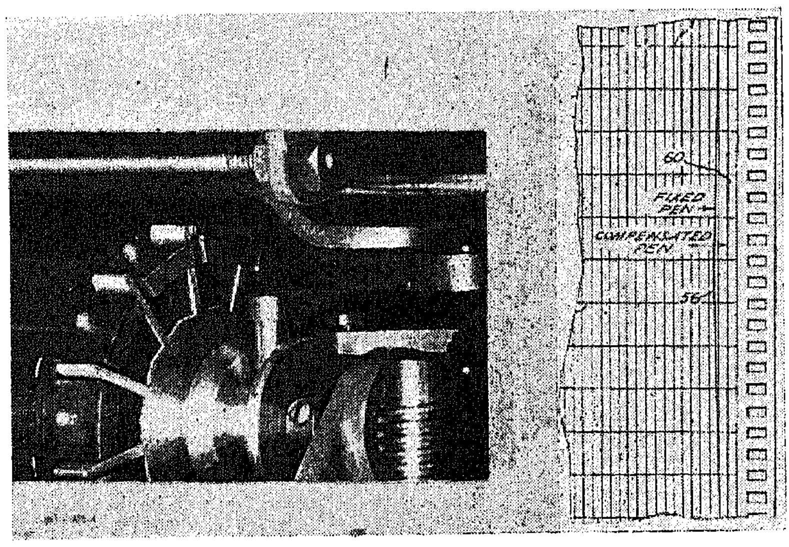

Figure 8 (right). Humidity compensator. To the left is shown actuating spider positioned axially by the chart. The chart shows performance with increase, then decrease, of humidity

objectionable because it does not average out, hence is a possible source of error. Such so-called "zero offset" of "zero error" in this amplifier is negligible with respect to the 2 microvolts which corresponds to 1/10 division on the chart or 0.001 ohm.

The circuit of the balancing amplifier is shown in Figure 10. The five stages of amplification are contained in three envelopes by the use of two twin triodes.

Many things had to be done to reduce the zero offset to a negligible value. One of them, evident from this circuit, is the use of direct current on the heaters. Many details in the amplifier are the same as those described in an earlier paper.[11]

The resistance-capacitance network at the left provides filtering to exclude a-c noise and provides damping to combat the tendency of the balancing system to overshoot the balance point.

Bridge Circuit

Figure 11 shows the bridge circuit. Connected to the slide-wire at the left is the balancing amplifier consisting of the

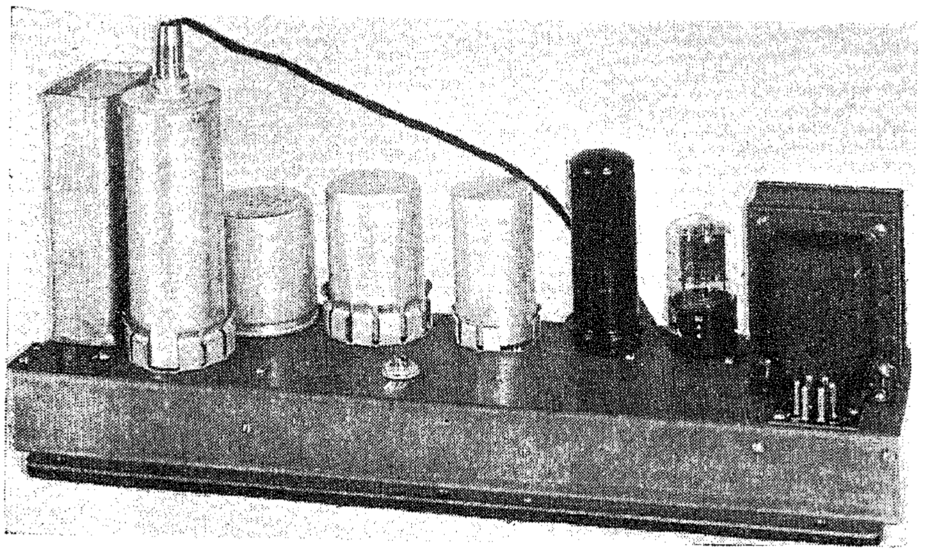

Figure 9. Balancing amplifier

Energization of the vibrator through its top assists in reducing zero offset to a negligible value

Table I. Corrections for Slide-Wire and Decades

The 10-Ohm Resistors Form a Decade which Deviates from its Average by only 0.0006 Ohm

Slide-wire											
0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	reading in ohms
0	−0.6	−0.8	−1.3	−1.8	−2.3	−3.0	−3.0	−3.8	−4.3	−4.3	correction in milliohms
One-Ohm Decade											
0	1.	2.	3.	4.	5.	6.	7.	8.	9.		reading in ohms
0	+1.1	+1.7	+2.2	+2.7	+3.2	+3.2	+3.2	+2.3	+2.8		correction in milliohms
Ten-Ohm Decade											
0	10	20	30	40	50	60	70	80	90		reading in ohms
0	+0.6	+0.2	−0.4	+0.2	−0.4	0.0	+0.3	0.0	+0.6		correction in milliohms

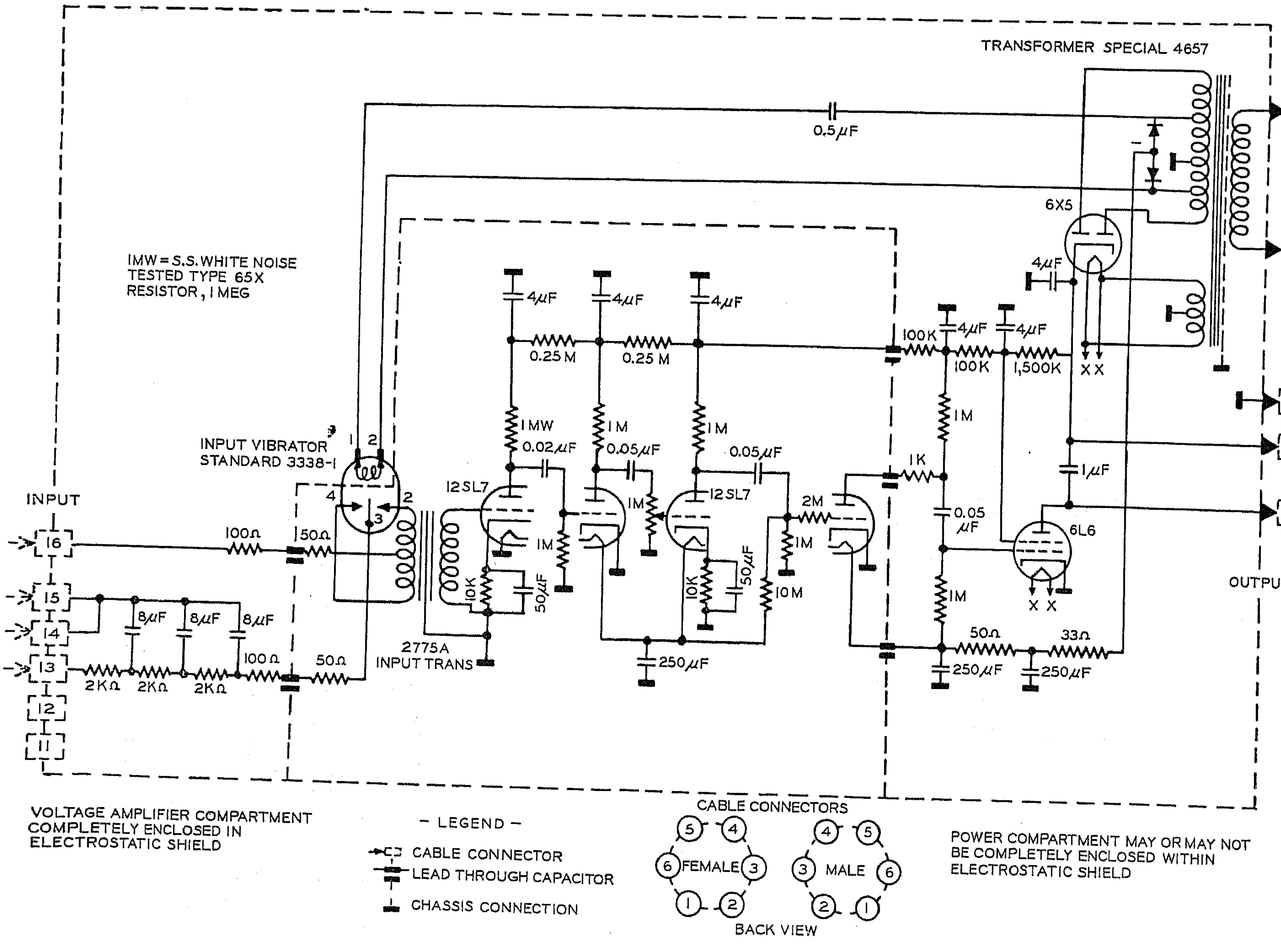

Figure 10. Circuit of balancing amplifier. An extra stage provides ample amplification

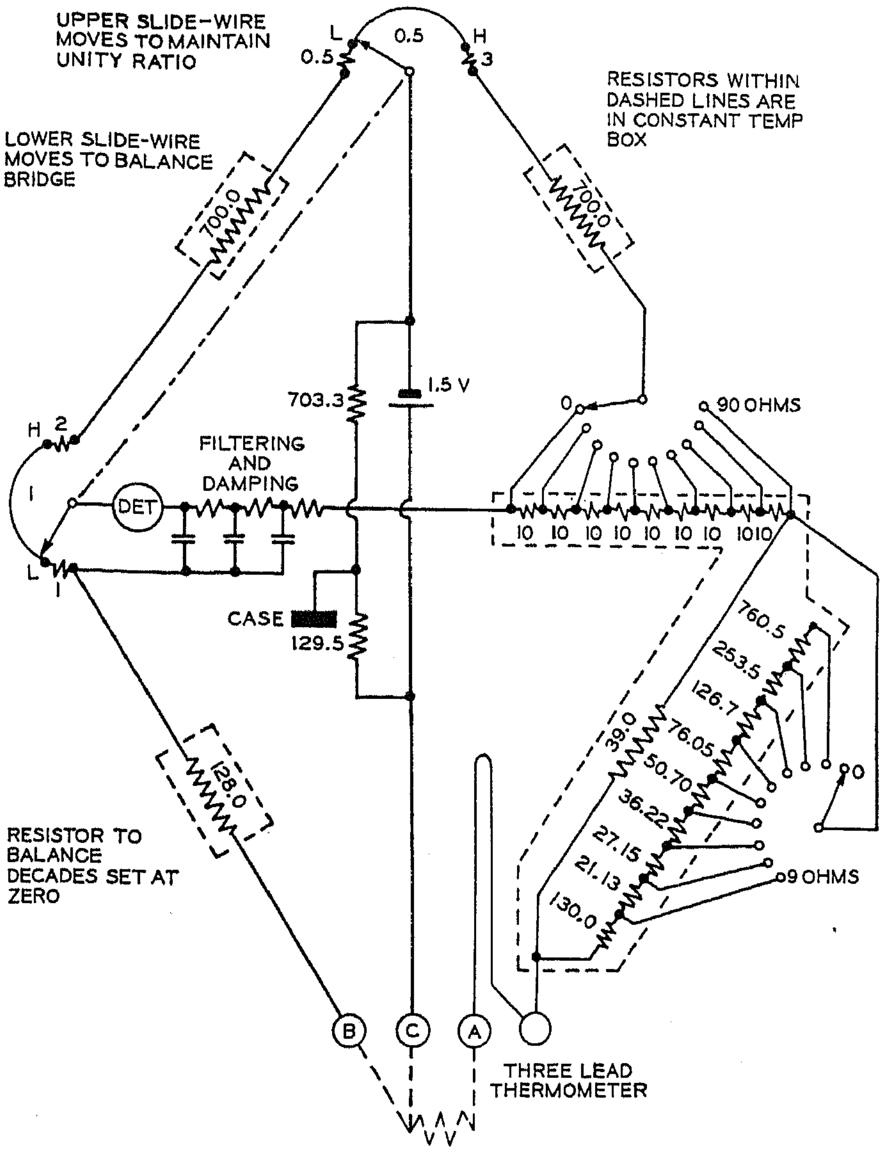

Figure 11 (above). Bridge circuit. The thermometer being in the same arm as the decades, a substitution type of measurement is made

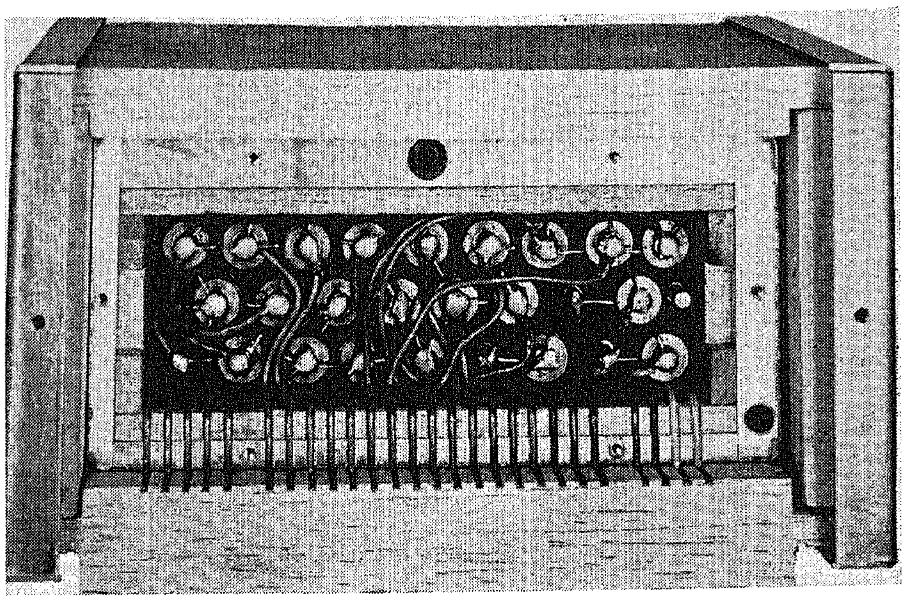

Figure 12 (above). Interior of constant temperature box

Fifty-five degrees centigrade was selected for the control point because the temperature inside of the recorder may rise 15 degrees centigrade above a possible ambient of 40 degrees centigrade

detector and the network for filtering and damping just mentioned.

The effect of contact resistance in the 10-ohm decade is suppressed by a factor of five by placing it in a relatively high-valued ratio arm. The effect of contact resistance in the 1-ohm decade is suppressed by a factor of six or more by the use of a shunted type of decade as in Mueller's hand-balanced bridge.

The thermometer being in the same arm as the decades, a substitution type of measurement is made. This gives constant sensitivity, allows the use of a simple "Wagner" ground and avoids the need for checking and adjusting the ratio arms specifically. Any change in the ratio arms is indicated by a change in the zero reading and simply requires a new zero correction.

The critical resistors within the dashed lines are located in a constant temperature box. Figure 12 shows its interior.

Constancy to 0.001 ohm in 100 ohms or 1 part in 100,000 was achieved by temperature control to 1 degree centigrade and reduction of temperature dependence in the resistors to 1 part in 100,000 per degree centigrade. Fifty-five degrees centigrade was selected for the control point because the temperature inside of the recorder may rise 15 degrees centigrade above a possible ambient of 40 degrees centigrade. The critical resistors were made to have their minimum temperature dependence in the region of 55 degrees centigrade by making each resistor partly of manganin and partly of copper, in series combination.

The Slide-wire

Figure 13 shows the slide-wire and its drive. A single moulded disk carries both

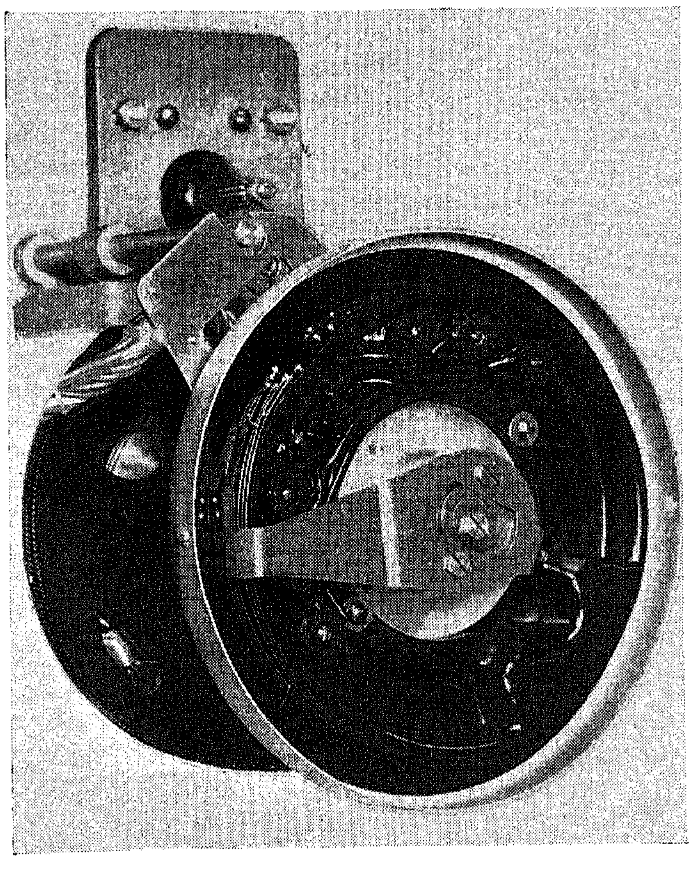

Figure 13 (left). Slide-wire and its drive

The slide-wire is continuous and so places no limit on the sensitivity of the recorder. It is housed in a magnetic shield of which only the back section is shown

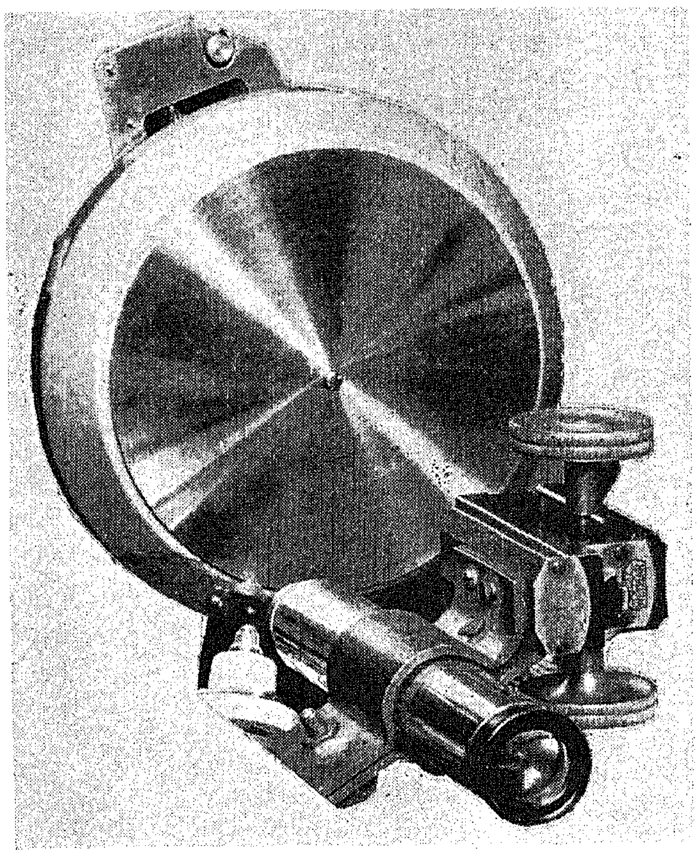

Figure 14 (right). Setup for precise checking

Lines on the disk fastened to the shaft are observed with reference to an index fastened to the slide-wire shield. A reticule in the microscope permits reading to 0.001 inch or 1/15,000th of the slide-wire travel of the slide-wires which appear in the bridge circuit. The disk is housed in a highly permeable magnetic shield of which only the back section is shown. Each slide-wire is continuous (not wound on a mandrel) and so places no limit on the sensitivity of the recorder.

Setup for Precise Checking

For ordinary use a sensitivity to about 1/1000th of the slide-wire span or 0.001 ohm is adequate, but for checking purposes more sensitivity was desired. The resistors in the recorder were intercompared by the so-called step-up method. In this method each 1-ohm step is compared to the slide-wire and each 10-ohm step is compared with the sum of the nine 1-ohm steps plus the slide-wire. Since the final calibration involves the sum of ten or more readings it was desirable to make each of these ten readings to one extra figure or to 0.0001 ohm.

It was found possible to do this with the setup shown in Figure 14. Lines on the disk, which is fastened to the slide-wire-contact shaft, are observed with reference to an index fastened to the slide-wire shield. A reticule in the microscope permits readings to 0.001 inch or 1/15,000th of the travel of the slide-wire contact. With the balancing amplifier set to optimum gain, readings were found to be reproducible to a value approaching 0.001 inch or 0.0001 ohm.

With these microscope readings it was possible to calculate errors and corrections for the 1-ohm and 10-ohm decades based on the average of the 10-ohm resistors.

These decade corrections are given in Table I. Apparently in the several years since these resistors were made and adjusted, the resistors making up the 1-ohm decade increased with respect to the 10-ohm resistors. The 10-ohm resistors still form a decade which deviates from its average by only 0.0006 ohm. The question might be asked whether their average had changed. Independent measurements with Standards of the National Bureau of Standards showed a drift downward in their resistance of about 0.005 per cent.

It is not necessary to know the exact amount of drift to use the recorder for precise temperature measurements with a platinum resistance thermometer. This is so since the Callendar formula[1] and the Van Dusen formula[2] for the platinum thermometer give the temperature, not as a function of its absolute resistance, but as the ratio of its resistance to its resistance at the ice point. Temperature, therefore, can be determined using resistance values measured in international ohms or absolute ohms or nonstandard ohms, provided only that the ice-point resistance is determined in terms of the same unit.

Summary

An attempt has been made to describe those features of a recorder which permit its unattended operation with a precision approaching 0.001 ohm or 0.01 degree centigrade for a 25-ohm platinum resistance thermometer.

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Discussion

C. R. Witschonke (American Cyanamid Company, Bound Brook, N.J.): The beautifully designed resistance recorder described by Mr. Williams greatly extends the accuracy in temperature measurements available for routine use by relatively unskilled personnel. Dr. Stull's original publications in this field interested us and subsequently several resistance recorders were constructed which have been applied to research and production materials. An article[2] has been printed briefly outlining one of these recorders.

Two different resistance recorders were developed. Both recorders used 10-degree-centigrade spans with 1/2-degree-centigrade overlap at each end and automatic range changing. The 124-ohm platinum resistance elements, for which these instruments were designed, were interchangeable.

The 24-range recorder, designed for laboratory research, used 5-stage amplification of a chopped d-c bridge signal and direct current on the heaters as in the Leeds and Northrup model. Tuned feedback circuits were included in addition, however, in order to give relatively narrow band-pass amplifier characteristics. This resulted in more than ample sensitivity for the 1.7-mil thermometer current operation. In fact, a change of 0.002 to 3 ohm (equivalent to 0.004 to 6 degrees centigrade) was detected easily by the recorder. The ingenious microscope technique described by Mr. Williams was not used so that it can be said that the stability was only as good as could be read on the engraved scale attached to the recorder, or about ±0.005 degree centigrade.

The second 4-decade recorder was designed for routine measurements. Only three stages of amplification of the a-c Wheatstone bridge signal were used. A thermometer current of 4 mils yielded sufficient signal for a temperature sensitivity of better than 0.01 degree centigrade. The stability was of the order of ±0.01 degree centigrade. The temperature increase due to the 4-mil current amounted to about 0.05 degree centigrade, but this effect was quite reproducible so that it was corrected for in the calibration of the instrument.

A novel feature of both of these recorders was that temperatures were read directly from the scale for all the decades. This was accomplished by changing the ratio arms for each decade in order to compensate for the slight nonlinearity in the temperature coefficient of platinum. Thus two resistances were accurately calibrated for each decade and replacement thermometers were constructed to have identical temperature coefficients and ice-point resistances within the desired accuracy of ±0.01 degree centigrade. Temperature coefficients were found to be amazingly reproducible when the highest purity platinum available was used and the duplication of ice-point resistances to 0.02 ohm has also proved to be practical. It is felt that the convenience of reading temperatures directly more than compensates for the complications involved. This is especially true when only a limited number of decades are involved. The method probably would be impractical for a span of more than 50 decades, however.

In the development of the recorders we were greatly aided by the Thwing-Albert Instrument Company and our electronics consultant, Mr. C. W. Warren.

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Daniel R. Stull (Dow Chemical Company, Midland, Mich.): It is well to examine periodically one's tools in the light of recent advances. As has been indicated, the International Temperature Scale from the boiling point of oxygen to the melting point of antimony is defined by international agreement in terms of a platinum resistance thermometer. It is, therefore, appropriate that precision measurement of such physical properties as freezing point, boiling point, vapor pressure, and so forth, be made with a platinum resistance thermometer. Previously, use of a platinum resistance thermometer tied a person to a measuring bridge. Use of skilled help to operate bridge dials probably does not utilize the individual's skill at anywhere near its maximum usefulness. The fully automatic recorder described here frees that individual to apply his talent and skill to problems of greater importance than that of seeking bridge balance and recording it.

A. J. Williams, Jr.: Dr. Stull's discussion explains very concisely the need for a recorder of the general type described in the paper.

Dr. Witschonke's discussion also indicates his appreciation of this need. His recorders were similar in many ways to the recorder described in the paper; for example, they had slide-wire spans of approximately 10 degrees centigrade and the recorder of greatest precision had direct current on the heaters of the critical amplifier tubes.

Dr. Witschonke has stated the important pros and cons on the direct-reading feature. Because we wished to cover the full range of the platinum resistance thermometer without risking the introduction of an additional error, we decided to build the simpler type of recorder which reads in resistance. The computation of temperature from resistance need not introduce any appreciable error.

The Fundamental Accuracy of Single-Phase 3-Wire Metering

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Synopsis: As a result of constantly increasing loads, 3-wire single-phase service is rapidly replacing the original 2-wire domestic service. This change has been accompanied by continuous improvement in the range and performance of the meters. In view of these developments, it is appropriate to analyze the bases upon which the fundamental correctness of 3-wire single-phase metering rests and which have been responsible for its universal acceptance for many years. The analysis in this paper shows that this acceptance is well founded, that the probable error of this form of metering can be calculated readily, and that it is usually negligible.

THE domestic consumption of electricity has increased constantly since the introduction of the a-c system. The past two decades have witnessed the general change from 2-wire to 3-wire service. While the 3-wire meter accounted for only 13 per cent of the single-phase watt-hour meters manufactured in 1925, this percentage had changed to 80 per cent in 1950. During this same 25-year period the average annual energy consumption per residential customer increased from 400 to 1,800 kilowatt-hours.

This period also has witnessed continuous improvement in performance and accuracy of the watt-hour meter. The overload capacity has been approximately tripled. Accuracy has been improved for load, voltage, and temperature variations. Meters have been protected against damage by lightning surges. Similarly, 3-wire current transformers also have been improved in accuracy and load range.

In view of these developments, an up-to-date analysis of the reasons for the fundamental accuracy of the single-element 3-wire metering circuit is appropriate. Application of the theory of probability in a rigorous manner shows why the accuracy of this method is well within commercial limits.

Outline of Method of Analysis

The approach of this paper is:

1. To calculate the error in terms of possible unbalances and circuit constants.
2. To determine the probability of all possible unbalances.
3. To evaluate the most probable error by a summation of the products of error and probability.
4. To evaluate the effect of simplifying assumptions made in analyzing the accuracy of registration.

Since the object is to analyze the fundamental accuracy of the circuit, effects, depending on the meter's calibration, load curve, balance of windings, voltage curve, and so forth, are not considered. (It is well known that the modern meter's performance is very good on these points.)

The Effect of Unbalanced Voltage on the Accuracy of a 3-Wire Meter

In accordance with Blondel's Theorem, two meter elements are required for accurate measurement of power in any 3-wire circuit. However, if the voltages from lines to neutral in a single-phase 3-wire circuit are equal, a single element with two current windings will meter the power correctly. If the voltages are slightly unbalanced, as is usually the case, very small errors result, providing the load also is unbalanced. Ordinarily, of course, unbalanced load is the only cause of unbalanced voltage.

Such a scheme is shown in Figure 1. If the loads from each line to neutral are equal, the voltages will be equal and the energy will be measured correctly. However, if the loads are not equal, the voltages will become unbalanced because of the unequal voltage drops in the transformer secondary and in the lines. The combination of unequal loads and unbalanced voltages causes slight errors.

The magnitude of the error is developed in Appendix I in terms of currents, voltages, and impedances. The error may be expressed as the product of half the unbalance in voltage times the power component of the neutral current. (Throughout the paper the dot product of two vectors such as $\mathbf{E}\cdot\mathbf{I}$ indicates $EI \cos \theta$ where θ is the angle between the vectors.)

$$\text{Error} = \left(\frac{\mathbf{E}_{10}-\mathbf{E}_{02}}{2}\right)\cdot\mathbf{I}_0$$

This is a general expression, independent of the number or size of the loads on the circuit, or the transformer and line impedances.

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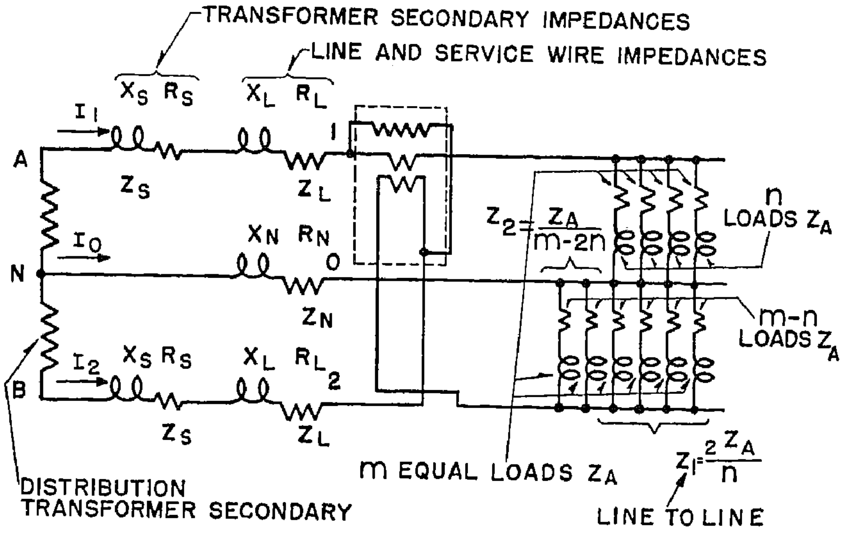

Figure 1. Circuit used for analyzing the effect on registration when a customer's load is unbalanced

This expression for error is of limited usefulness because the unbalance of load is not usually known. To facilitate the analysis, it is assumed for the moment that there is a single customer connected to the distribution transformer as shown in Figure 1. This customer has m equal loads, of which n are connected from one line to neutral. Appendix II shows that the percentage error may then be expressed as

$$\text{Per-cent error} = 100\,\frac{(R_S+R_L+2R_N)}{2R_A}\times\frac{(m-2n)^2}{m} \qquad (11)$$

$$= 100\,\frac{(R_S+R_L+2R_N)}{2R_2}\,\frac{W_1}{W_2} \qquad (16)$$

$$= \frac{100\,W_L}{W_1} \qquad (19)$$

where

R_S = one-half the transformer secondary resistance
R_L = resistance of one line wire and one service wire
R_N = neutral resistance
R_A = resistance of each load unit
m = total number of load units
n = number of load units connected from one line to neutral
$R_2 = R_A/(m-2n)$
W_1 = total power consumed
W_2 = power consumed by unbalanced portion of the load
W_L = total additional line and transformer losses caused by the unbalance

Equations 11, 16, and 19 all show that the error is plus. The reason for this is apparent from physical considerations. A load from one line to neutral reduces the voltage between that line and neutral because of impedance drops in the line transformer secondary. However, the voltage applied to the meter is the line-to-line voltage which has decreased only half as much on a percentage basis. The error is, therefore, plus for the condition of a single customer connected to the distribution transformer.

Equation 11 expresses the error in terms of the circuit resistances and the unbalance in the load. Since m represents the total number of load units, and n the number connected from one line to neutral, the number of balanced load units is represented by $2n$, and the number of unbalanced load units by m-$2n$.

It will be observed that for a single load unit of resistance R_A, the per-cent error is $100\,(R_S+R_L+2R_N)/2R_A$ since $m=1$ and $n=0$. This might be called the basic error and, as would be expected, depends on the ratio of circuit resistance to load resistance. For other values of m and n the basic error is multiplied by $(m-2n)^2/m$ or $[(m-2n)/m]\,(m-2n)$. $(m-2n)/m$ represents the portion of the load which is unbalanced and $m-2n$ the number of unbalanced load units.

Equation 16 expresses the error in terms of the ratio of circuit resistance to unbalanced load resistance times the ratio of unbalanced power to total power.

It is also apparent from equations 11 and 16, as would be expected, that the error is proportional to the ratio of transformer and line resistance to load resistance.

Equation 19 is very informative, since it shows that the error is always equal to the total additional losses in the line and transformer caused by the unbalanced portion of the load. For example, the transformer's secondary copper loss and line losses for a balanced load of 240 watts may total 0.2 watt. If an unbalanced load of 240 watts causes these losses to rise to 0.7 watt, the increased loss attributable to the unbalance is 0.5 watt. According to equation 19 the error in registration for the unbalanced condition therefore would be 0.5 watt or slightly less than 0.2 per cent. In other words, the meter measures the load power plus the increase in transformer and line losses caused by unbalance.

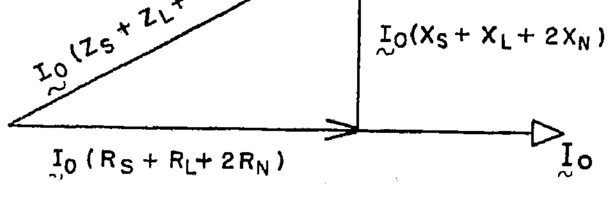

Figure 2. Vector diagram of neutral current and associated voltage drops

Probability of Unbalanced Load

Equations 11, 16, and 19 are sufficient to calculate the registration for any given load unbalance.

The next step is to calculate the probability of unbalance, assuming that, when a load is applied, there is an equal probability of its being connected to either line.

Appendix III shows that if there are m equal load units the probability of n units being connected from one line to neutral is $m!/2^m n!(m-n)!$ As an example, if there are 20 loads ($m=20$) the chances of complete unbalance ($n=0$ and $n=m$) and $1/2^{19}=1/524{,}288$ or not quite two in a million.

Probable Error

The probable error for a given unbalance is the error for that unbalance multiplied by the probability of that unbalance. The total most probable error is the summation of these probable errors for all possible unbalances. Equation 39 developed in Appendix IV shows that the total most probable error in per cent is $100\,(R_S+R_L+2R_N)/2R_A$

where

R_S = one-half the transformer secondary resistance
R_L = resistance of one line wire and one service wire
R_N = neutral resistance
R_A = resistance of each load unit

In other words, the probable percentage error is that which is caused by a single load unit being connected from line to neutral. Whether there are 10, 20, or 100 such load units to be connected, the probable percentage error remains that of a single unbalanced unit. This is somewhat surprising, since the maximum possible percentage error increases with the number of load units. However, the probability of encountering complete unbalance of a large number of units is so remote that the probable percentage error remains that caused by the unbalance of a single unit.

Another conclusion, again fairly obvious, is that the probable error is dependent on the ratio of transformer and

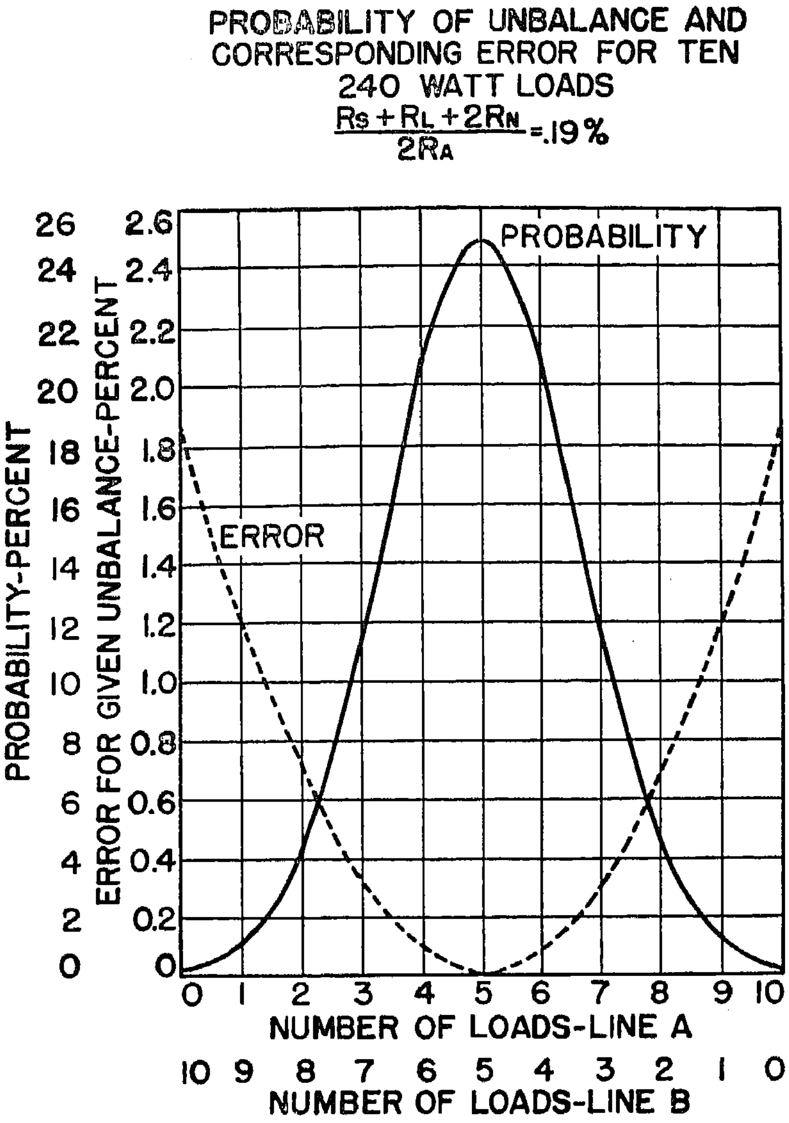

Figure 3 (left). Probability of unbalance and corresponding error

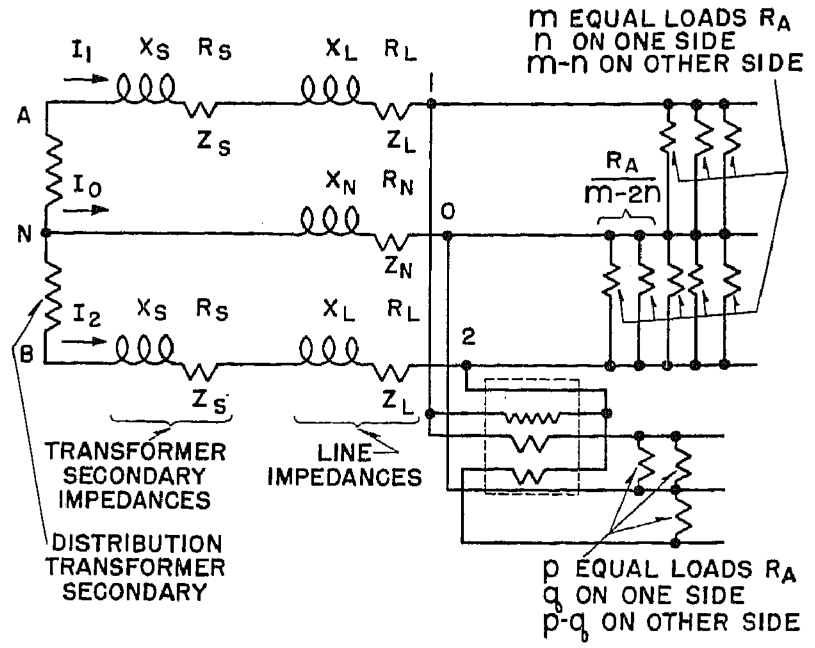

Figure 5 (right). Circuit used for analyzing the effect of other customers' unbalanced load on a given customer's registration

line resistance to load resistance. Low supply resistance (which is equivalent to high capacity) reduces errors. Similarly, the smaller the size of the assumed load unit, the smaller the errors.

Numerical Example

At this point it is well to consider some numerical values. The assumed size of the 2-wire load unit is not subject to precise calculation or prediction, since loads vary from electric clocks to ironers, a span of 2 to 1,500 watts. Units in the upper range, however, are relatively rare compared to the number of lamps and appliances in the 50- to 240-watt span. Assuming a basic load value of 240 watts or $R_A=60$ is probably pessimistic.

If the distribution transformer is rated at 10 kva, the copper loss in the secondary at rated load is probably approximately 90 watts.

$10{,}000/240=41.67$ amperes
$41.67^2\times 2R_S=90$
$R_S=90/3{,}470=0.026$ ohm

The resistance of number 4 wire is about 0.25 ohm per 1,000 feet. Assuming 150 feet of number 4 line wire, $R_L=R_N=0.038$ ohm. The service wire may be 75 feet of number 6 or $0.075\times 0.394=0.030$ ohm to be added to R_L and R_N. Substituting in equation 39: $R_S=0.026$, $R_L=0.068$, $R_N=0.068$, and $R_A=60$.

$$\frac{100(R_S+R_L+2R_N)}{2R_A}\text{ per cent}$$
$$=\frac{100(0.026+0.068+0.136)}{2\times 60}\text{ per cent}$$
$$=0.19\text{ per cent}$$

The probable error thus is 0.19 per cent. Figures 3 and 4 illustrate the factors involved for the same impedance values but for a total of ten loads on the line ($m=10$). (While each variable is shown by a smooth curve for easier visualization, only the points corresponding to integral load values are significant.)

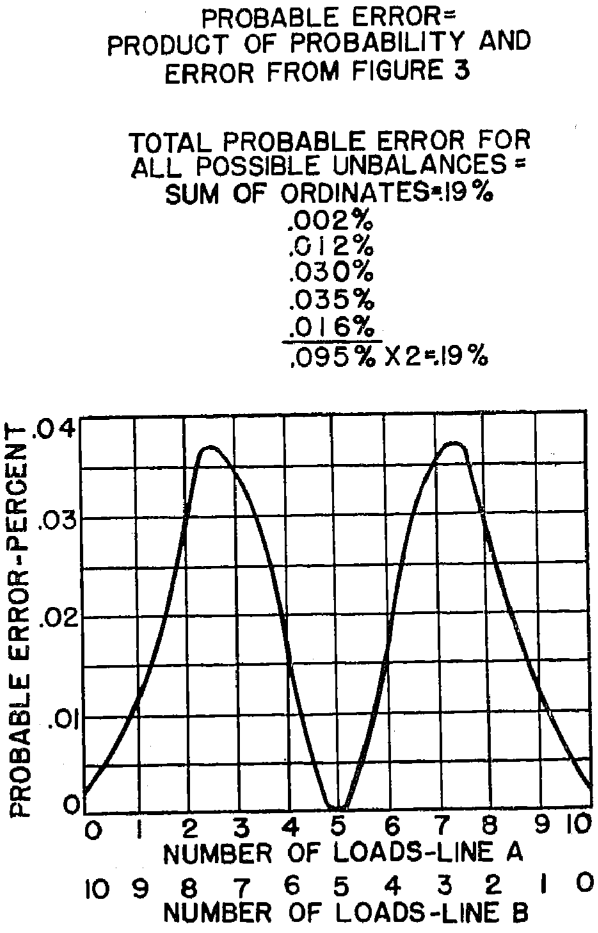

Figure 4. Probable error caused by unbalanced loads

The solid curve of Figure 3 shows the probability of a given number of loads being connected to one line. This curve is very similar to the "normal" or Gaussian distribution curve which frequently is used in quality control analysis. It is interesting to note that the chances are almost one out of four that the ten loads will be balanced. Also, the chances are almost two out of three (two-to-one odds) that ten loads will be split either five-five or six-four.

The dotted curve of Figure 3 shows the error in per cent for each condition of load distribution.

Figure 4 represents the product of the two curves of Figure 3. Therefore, the ordinate at each point represents the probable error due to that particular division of load. The sum of these ordinates is, therefore, the total most probable error for all possible unbalances. This sum equals 0.19 per cent as shown on Figure 4, thus checking equation 39.

If the basic load unit is assumed to be smaller than 240 watts, the probable error decreases proportionally. Table I shows the probable error caused by line and service wire resistances, assuming a basic load unit of 240 watts.

The transformer introduces a slight additional error, but in the numerical example the probable error caused by a 10-kva transformer is only 0.02 per cent so it can be disregarded for all practical purposes.

Effect of Unequal Loads

The solution presented involved the assumption that all load units were equal.

Table I. Probable Error for 240-Watt Load Unit

Wire Size, in Service Run or Feeder	Most Probable Error, Per Cent per 100 Feet of Run
Number 10	0.25
Number 8	0.16
Number 6	0.1
Number 4	0.06
Number 2	0.04
Number 0	0.025
Number 00	0.02
Number 0000	0.012

This obviously is not the case, but no other assumption lends itself readily to analysis.

However, it seems logical to assume that the probable error at any given time will not exceed the value calculated for a load unit equal to the largest unit in service at that time. For example, if the largest single load connected line to neutral is a 1,000-watt toaster, a probable error calculation based on all equal loads of this size is, if anything, pessimistic. Offsetting such a condition is the fact that for extended periods the largest single load in service is only 100 watts or less. Averaging the periods of service of the various loads, therefore, would probably give a load unit value of considerably less than the 240 watts assumed in the example.

Further indication that 240 watts is, if anything, a pessimistic value for each load unit is derived from annual residential kilowatt-hour consumption data. For the average residential consumption of 1,800 kwh per year, the total of all the loads of a residence average only 300 watts on a 16-hour-a-day basis.

Effect of Other Customers' Loads

The foregoing analysis is valid for the error of a given customer caused by his own unbalance. However, if there are a number of customers on the line, there is an additional error dependent on the load balance of the other customers.

Assuming, for the moment, that there are a number of customers on a line and that a given customer has a single load unit connected between line 1 and neutral, there are three possibilities regarding the rest of the load on the lines:

1. Remainder of load also heavier on line 1.
2. Remainder of load balanced.
3. Remainder of load heavier on line 2.

The effect of these loads on this customer's registration is to give plus errors in case 1, no error in case 2, and minus errors in case 3. The most probable error caused by the other loads on his registration is zero since the effect is just as likely to be minus as plus.

In other words, if any customer adds an unbalanced load to a circuit with a given load distribution, his registration will be high if his load aggravates an existing unbalance, and low if it decreases an existing unbalance.

This condition is analyzed in Appendix V. The probable voltage unbalance is determined for any total load on the circuit. The effect of this unbalance on a given customer's registration is then calculated. Figure 5 shows the circuit which applies to this analysis.

The ratio $E=(R_S+R_L+2R_N)/2R_A$ is again what might be called a basic error. Assuming that a given customer has a single load unit connected between line 1 and neutral, the basic error is that caused by another single load unit being connected by some other customer. This error may be either plus or minus. For more general conditions this basic error must be multiplied by two factors. The first, M, depends on the number and probable distribution of other customers' loads. The second, F, depends on the number and probable distribution of the given customer's loads. The total most probable error caused by other customers' loads is $\pm E\times M\times F$ as given by equation 50 in Appendix V.

This equation is now applied to an example. The same values are assumed as in the previous example (except that the customer's service wire resistance is omitted since other customer's loads are not carried by it).

$R_S=0.026$, 10-kva transformer
$R_L=0.038$, 150 feet of number 4 wire (service wire is not involved in this condition)
$R_N=0.038$
$R_A=60$
$m=10$, number of other customers' loads on the circuit
$p=3$, number of loads the given customer has on the circuit

Table II. Values of Multiplier $\dfrac{mm!}{2^m\left(\dfrac{m}{2}!\right)^2}$ **or** $\dfrac{m!}{2^{m-1}\left(\dfrac{m-1}{2}!\right)^2}$

m	Multiplier M
1,2	1
3,4	1.5
5,6	1.875
7,8	2.188
9,10	2.461
19,20	3.524
39,40	5.015
99,100	7.959

Table III. Values of Factor F for Various Values of p

Number of Customers Load Units (p)	Factor F
1	1
2	0.5
3	0.5
4	0.375
5	0.375
6	0.312
7	0.312
8	0.273
9	0.273

From equation 39

$$E=(R_S+R_L+2R_N)/2R_A=0.14/120$$
$$=0.117 \text{ per cent}$$

From Table II

for $m=10$ $\quad M=2.46$

From Table III

for $p=3$ $\quad F=0.5$

Total most probable error $=E\times M\times F=$ 0.117 per cent $\times$ 2.46 $\times 0.5=\pm 0.144$ per cent.

Table IV gives total most probable errors assuming (pessimistically) that the transformer is operating at full load $(p+m)=42$.

Table IV shows that, if the customer has more than one load unit connected, the probable error is reduced rapidly because the error caused by other customers applies only to the unbalanced portion of his load. For example, the probable error is ± 0.3 per cent if the customer has two loads connected, compared to ± 0.6 per cent if he has only one load connected.

If the basic load unit is 100 watts instead of 240 watts, the probable error is reduced to about two-thirds of the values just given for given values of total load. It should be noted therefore that this component of the error is rarely as great as 0.3 per cent and tends to average zero.

Effect of Transformer and Line Reactance

One other factor which has been neglected in the foregoing analysis is reactance of transformer and line. Resistive loads cause voltage drops in line and transformer resistances. Likewise, the reactive component of the load causes a voltage drop of the same phase position in the transformer and line reactances. The effect is similar to that described in the foregoing and may be analyzed in the same way. Transformer reactances are normally approximately the same as the resistance, while line reactance is approximately 0.15 ohm per 1,000 feet. The net effect for most conditions is, therefore, somewhat comparable. The errors caused by reactive loads are equally likely to be plus or minus.

Effect of Line-to-Line Loads

All calculations and conclusions to this point have neglected one very important consideration, namely, the existence of 240-volt loads, such as ranges, hot water heaters, clothes driers, and some motors. These loads are always balanced and, therefore, measured accurately, reducing the probable errors a great deal. The effect is twofold:

1. Total registration is increased, reducing the percentage errors.
2. Since the capacity of the transformer is limited, the possible unbalanced load cannot be as great as assumed in the examples given.

The effect of 240-volt loads in minimizing probable errors is considerable since most of the heavier load units are in this category.

Polyphase Metering

The foregoing analyses of the accuracy of the 3-wire single-phase meter are applicable to a large extent to the 3-wire elements of polyphase meters. Two meters in common use incorporate such elements, namely, the 2-element meter for 3-phase 4-wire delta circuits, and the "$2^1/_2$ element" meter for 3-phase 4-wire circuits. The general assumption of balanced voltage also applies to these meters. The same method of analysis can be used and the conclusions may safely be expected to be comparable; that is, errors probably will not exceed tenths of a per cent. For polyphase service a larger percentage of the load normally will be balanced than for single phase.

Conclusions

A thorough analysis of the probable loading of single-phase 3-wire circuits and the effects on voltage balance shows that the registration of the single-element 3-wire meter is extremely accurate. For unbalances created by a customer, the probable error is of the order of +0.2 per cent even under pessimistic assumptions as to load size. Probable unbalance of other customers' loads, again with extremely pessimistic assumptions, results in an error of the same order of magnitude and this tends to average zero. In general, it seems safe to conclude that over a period of a month, the errors caused by unbalanced loads will be 0.2 per cent or less. While such high accuracy has generally been assumed, the analyses provide a sound basis for this conclusion.

Appendix I. Calculation of Error Caused by Unbalanced Load

$$\text{True power} = \mathbf{E}_{10}\cdot\mathbf{I}_1 + \mathbf{E}_{20}\cdot\mathbf{I}_2 \qquad (1)$$

$$\text{Measured power} = \mathbf{E}_{12}\cdot(\mathbf{I}_1 - \mathbf{I}_2)/2 \qquad (2)$$

$$\text{Error} = \text{true power} - \text{measured power}$$

$$= \mathbf{E}_{10}\cdot\mathbf{I}_1 + \mathbf{E}_{20}\cdot\mathbf{I}_2 - \mathbf{E}_{12}\cdot(\mathbf{I}_1 - \mathbf{I}_2)/2 \qquad (3)$$

But $\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_0 = 0$, so $\mathbf{I}_2 = -\mathbf{I}_1 - \mathbf{I}_0$. Also $\mathbf{E}_{12} = \mathbf{E}_{10} - \mathbf{E}_{20}$. Substituting these values in equation 3

$$\text{Error} = \mathbf{E}_{10}\cdot\mathbf{I}_1 - \mathbf{E}_{20}\cdot\mathbf{I}_1 - \mathbf{E}_{20}\cdot\mathbf{I}_0 - \mathbf{E}_{10}\cdot\left(\frac{2\mathbf{I}_1 + \mathbf{I}_0}{2}\right) + \mathbf{E}_{20}\cdot\left(\frac{2\mathbf{I}_1 + \mathbf{I}_0}{2}\right) = -\mathbf{E}_{10}\cdot\left(\frac{\mathbf{I}_0}{2}\right) - \mathbf{E}_{20}\cdot\left(\frac{\mathbf{I}_0}{2}\right) = -(\mathbf{E}_{10} - \mathbf{F}_{02})\cdot\frac{\mathbf{I}_0}{2} \qquad (4A)$$

$$= -\left(\frac{\mathbf{E}_{10} - \mathbf{E}_{02}}{2}\right)\cdot\mathbf{I}_0 \qquad (4B)$$

Equations 4(A) and 4(B) are general equations expressing the error resulting from unbalance of load and voltage.

Table IV. Most Probable Errors Caused by Other Customers' Unbalance

$E = (R_S + R_L + 2R_N)/2R_A = 0.117$ Per Cent. $(p+m) = 42$; Transformer Operating at Full Load. $R_A = 60$

Number of Given Customers Load Units (p)	Number of Other Customers Load Units (m)	Multiplier M	Factor F	Total Most Probable Error in Per Cent Caused by Other Customers' Unbalance =E×M×F
1	41	5.14	1	±0.60
2	40	5.02	0.5	±0.30
3	39	5.02	0.5	±0.30
4	38	4.89	0.375	±0.21
5	37	4.89	0.375	±0.21
6	36	4.75	0.312	±0.17
7	35	4.75	0.312	±0.17
8	34	4.61	0.273	±0.15
9	33	4.61	0.273	±0.15
10	32	4.47	0.246	±0.13
11	31	4.47	0.246	±0.13
12	30	4.34	0.221	±0.11

Appendix II. Determining Percentage Error

To determine the percentage error, it is assumed, for purposes of calculation, that: (1) a single customer is on the line; and (2) this customer's load consists of m identical impedance units, Z_A, of which n units are connected from line 1 to neutral.

From Figure 1

$$\mathbf{E}_{10} = \mathbf{E}_{AN} - \mathbf{I}_1(Z_S + Z_L) + \mathbf{I}_0 Z_N$$

$$\mathbf{E}_{20} = \mathbf{E}_{BN} - \mathbf{I}_2(Z_S + Z_L) + \mathbf{I}_0 Z_N = -E_{AN} + \mathbf{I}_1(Z_S + Z_L) + I_0(Z_S + Z_L + Z_N)$$

$$(\mathbf{E}_{10} - \mathbf{E}_{02}) = \mathbf{I}_0(Z_S + Z_L + 2Z_N) \qquad (5)$$

From equations 4(B) and 5

$$\text{Error} = \mathbf{I}_0\cdot\mathbf{I}_0(Z_S + Z_L + 2Z_N)/2$$

$$= I_0^2(R_S + R_L + 2R_N)/2 \text{ from Figure 2} \qquad (6)$$

$$\text{Per-cent error} = \frac{100 I_0^2(R_S + R_L + 2R_N)/2}{\mathbf{E}_{12}\cdot(\mathbf{I}_1 - \mathbf{I}_2)/2}$$

$$= \frac{100 I_0^2(R_S + R_L + 2R_N)/2}{\mathbf{E}_{12}\cdot(2I_1 + I_0)/2} \qquad (7)$$

$$\text{But } \mathbf{I}_1 = \frac{E_{12}}{Z_1} \text{ and } \mathbf{E}_{12}\cdot 2\mathbf{I}_1 = \frac{2E_{12}^2 R_1}{Z_1^2} \qquad (8)$$

$$\mathbf{I}_0 = \frac{\mathbf{E}_{12}}{2Z_2} \text{ and } \mathbf{E}_{12}\cdot\mathbf{I}_0 \approx \frac{E_{12}^2 R_2}{2Z_2^2} \qquad (9)$$

Substituting these values in equation 7

$$\text{Per-cent error} = \frac{100(R_S + R_L + 2R_N)/2}{4R_1(Z_2/Z_1)^2 + R_2}$$

If all load units are equal, $R_2/Z_2 = R_1/Z_1$ or $R_2 = R_1(Z_2/Z_1)$.

$$\text{Per-cent error} = \frac{100(R_S + R_L + 2R_N)/2}{4R_1\left[\left(\frac{Z_2}{Z_1}\right)^2 + \frac{Z_2}{4Z_1}\right]} \qquad (10)$$

From Figure 1

$$\frac{Z_2}{Z_1} = \frac{Z_A}{m - 2n}\ \frac{n}{2Z_A} = \frac{n}{2(m - 2n)}$$

$$R_1 = 2R_A/n$$

Substituting these values in equation 10 and simplifying

Per-cent error =

$$\frac{100(R_S + R_L + 2R_N)/2}{2R_A\left[\left(\frac{n}{m - 2n}\right)^2 + \frac{n}{2(m - 2n)}\right]}$$

$$= \frac{100(R_S + R_L + 2R_N)(m - 2n)^2}{2mR_A} \qquad (11)$$

This may be expressed in several alternative forms.

$$\text{Let } W_1 = m\left(\frac{E_{12}}{2Z_A}\right)^2 R_A \approx \text{total power} \qquad (12)$$

$$W_2 \approx (m - 2n)\left(\frac{E_{12}}{2Z_A}\right)^2 R_A \text{ total unbalanced power} \qquad (13)$$

Then $(W_1 - W_2)$ = balanced power

From Figure 1

$$R_2 = R_A/(m - 2n) \text{ and } Z_2 = Z_A/(m - 2n) \qquad (14)$$

From equations 12 and 13

$$W_2/W_1=(m-2n)/m \qquad (15)$$

Substituting equations 14 and 15 in equation 11

$$\text{Per-cent error}=\frac{100(R_S+R_L+2R_N)}{2R_2}\frac{W_2}{W_1} \qquad (16)$$

From equations 9, 13, and 14

$$W_2=I_0^2R_2 \qquad (17)$$

Substituting equation 17 in 16

$$\text{Per-cent error}=\frac{100(R_S+R_L+2R_N)}{2}\frac{I_0^2}{W_1} \qquad (18)$$

The transformer and line losses may be compared for a given load under balanced and unbalanced conditions.

Let

W_B = losses caused when a given load is balanced
W_U = losses caused when the same load is unbalanced
$W_L=W_U-W_B$ = additional loss caused by the fact that the load is unbalanced

$$\text{Then } W_B=\left(\frac{E_{12}}{4Z_A}\right)^2(2R_S+2R_L)$$

$$W_U=\left(\frac{E_{10}}{Z_A}\right)^2(R_S+R_L+R_N)$$

where $\frac{E_{12}^2}{4Z_A}\approx\frac{E_{10}^2}{Z_A}$ (equal power under balanced and unbalanced conditions)

$$W_L=W_U-W_B=\left(\frac{E_{10}}{Z_A}\right)^2(R_S+R_L+R_N)-\left(\frac{E_{12}}{4Z_A}\right)^2(2R_S+2R_L)$$

But $\frac{E_{10}}{Z_A}=I_0$ and $E_{12}\approx 2E_{10}$

$$\text{So } W_L=I_0^2(R_S+R_L+R_N)-\frac{I_0^2}{4}(2R_S+2R_L)$$

$$W_L=I_0^2\left(\frac{R_S+R_L+2R_N}{2}\right)$$

Substituting in equation 18

$$\text{Per-cent error}=100\frac{W_L}{W_1} \qquad (19)$$

In other words, the increase in registration caused by the unbalanced load is equal to the additional transformer and line losses caused by the fact that a portion of the load is unbalanced.

Appendix III. Probability of Connecting *n* Units of Load Between One Line and Neutral when the Total Number of Load Units = *m*

The total number of combinations of m units taken n at a time $=C(m,n)=\frac{m!}{n!(m-n)!}$.

As an example, $n=0$ and $n=m$ represent complete unbalance, one equivalent to connecting all the loads on one line and the other equivalent to connecting all the loads on the other line.

$$C(m,0)=\frac{m!}{0!(m!)}=1$$

$$C(m,m)=\frac{m!}{m!\,(0!)}=1$$

The total possible number of loading combinations is the summation of the values for all possible values of n or

$$\sum_{n=0}^{m}C(m,n)=\sum_{n=0}^{m}\frac{m!}{n!(m-n)!} \qquad (20)$$

The binomial theorem states

$$(a+b)^m=\sum_{n=0}^{m}C(m,n)a^{m-n}b^n$$

$$(1+1)^m=\sum_{n=0}^{n=m}C(m,n)=2^m$$

The probability of the existence of any given combination of loading is, therefore, the ratio of the number of combinations giving this loading to the total possible combinations

P_n = probability of n loads on one specified

$$\text{side}=\frac{C(m,n)}{2^m}=\frac{m!}{2^m n!(m-n)!} \qquad (21)$$

Appendix IV. Calculation of Total Most Probable Error

From Appendix II, equation 11, the error is given by

Per-cent error

$$=\frac{100(R_S+R_L+2R_N)(m-2n)^2}{2mR_A} \qquad (11)$$

and the probability of n loads being connected one one side from Appendix III, equation 21, is

$$P_n=\frac{m!}{2^m n!(m-n)!} \qquad (21)$$

so that the probable error for any loading n is the product of equations 11 and 21.

Probable error (per cent) due to any given number n of loads on one line $n=0$ to $n=m$

$$=\frac{100(R_S+R_L+2R_N)(m-2n)^2m!}{2mR_A2^m n!(m-n)!} \qquad (22)$$

The total most probable error will be the summation of all the probable errors from $n=0$ to $n=m$.

Total most probable error, per cent

$$=\frac{100\,(R_S+R_L+2R_N)}{2mR_A}\frac{m!}{2^m}\times\sum_{n=0}^{m}\frac{(m-2n)^2}{n!(m-n)!} \qquad (23)$$

Total most probable error, per cent

$$=\frac{100(R_S+R_L+2R_N)}{2mR_A}\frac{m!}{2^m}\times\left[\sum_{n=0}^{m}\frac{m^2}{n!(m-n)!}-\sum_{n=0}^{m}\frac{4nm}{n!(m-n)!}+\sum_{n=0}^{m}\frac{4n^2}{n!(m-n)!}\right] \qquad (24)$$

This can be simplified by using the Binomial Theorem to obtain values for the summations inside the bracket. In the following development, equations 25 and 26 simplify the first summation, equations 27 to 34 the second summation, and equations 35 to 38 the third summation.

$$(a+b)^m=\sum_{n=0}^{m}\frac{m!a^{(m-n)}b^n}{n!(m-n)!}$$

If $a=b=1$, then

$$2^m=\sum_{n=0}^{m}\frac{m!}{n!(m-n)!} \qquad (25)$$

Multiplying by $m^2/m!$

$$\sum_{n=0}^{m}\frac{m^2}{n!(m-n)!}=\frac{m^2 2^m}{m!} \qquad (26)$$

Replacing m by $(m-1)$ in equation 25 gives

$$2^{m-1}=\sum_{n=0}^{m-1}\frac{(m-1)!}{n!(m-1-n)!}$$

$$=\sum_{n=0}^{m-1}\frac{(m-1)!(m-n)}{n!(m-n)!} \qquad (27)$$

$$=\sum_{n=0}^{m-1}\frac{m!}{n!(m-n)!}-\sum_{n=0}^{m-1}\frac{(m-1)!n}{n!(m-n)!} \qquad (28)$$

The upper limit may be extended to m by evaluating the term for $n=m$ and correcting for its value.

$$\text{If } n=m,\text{ then } \frac{m!}{n!(m-n)!}=1 \qquad (29)$$

$$\text{so that } \sum_{n=0}^{m-1}\frac{m!}{n!(m-n)!}=\left[\sum_{n=0}^{m}\frac{m!}{n!(m-n)!}\right]-1 \qquad (30)$$

$$\text{And if } n=m,\text{ then } \frac{(m-1)!n}{n!(m-n)!}=\frac{m!}{m!0!}=1 \qquad (31)$$

so that $$\sum_{n=0}^{m-1}\frac{(m-1)!n}{n!(m-n)!}$$

$$=\left[\sum_{n=0}^{m}\frac{(m-1)!n}{n!(m-n)!}\right]-1 \qquad (32)$$

Substituting equations 30 and 32 in 28

$$2^{m-1}=\left[\sum_{n=0}^{m}\frac{m!}{n!(m-n)!}\right]-1-\left[\sum_{n=0}^{m}\frac{(m-1)!n}{n!(m-n)!}\right]+1$$

Substituting equation 25 for the first term, this can be rewritten as

$$2^{m-1}=2^m-\sum_{n=0}^{m}\frac{(m-1)!n}{n!(m-n)!}$$

$$\sum_{n=0}^{m}\frac{(m-1)!n}{n!(m-n)!}=2^m-2^{m-1}=2^{m-1} \qquad (33)$$

Multiplying 33 by $4m/(m-1)!$ gives

$$\sum_{n=0}^{m}\frac{4mn}{n!(m-n)!}=\frac{4m2^{m-1}}{(m-1)!}=\frac{2m^2 2^m}{m!} \qquad (34)$$

Similarly, replacing m by $m-2$ in equation 25 gives

$$2^{m-2}=\sum_{n=0}^{m-2}\frac{(m-2)!}{n!(m-2-n)!}$$

$$=\sum_{n=0}^{m-2}\frac{(m-2)!(m-n)(m-1-n)}{n!(m-n)!} \qquad (35)$$

If $n=m-1, \quad m-1-n=0$
If $n=m, \quad m-n=0$

In either case equation 35 would be zero for $n=m-1$ or $n=m$ so that the upper limit of 35 can be extended to $n=m$ without changing its value. Expanding 35

$$2^{m-2}=\sum_{n=0}^{m}\frac{(m-2)![m(m-1)-n(m-1)-nm+n^2]}{n!(m-n)!} \qquad (36)$$

$$2^{(m-2)}=\sum_{n=0}^{m}\left(\frac{m!}{n!(m-n)!}-\frac{(m-1)!n}{n!(m-n)!}-\frac{(m-2)!mn}{n!(m-n)!}+\frac{n^2(m-2)!}{n!(m-n)!}\right)$$

Substituting equations 25 and 33 for the first two terms give

$$2^{(m-2)}=2^m-2^{m-1}-\sum_{n=0}^{m}\left(\frac{(m-1)!mn}{n!(m-n)!(m-1)}-\frac{n^2(m-2)!}{n!(m-n)!}\right)$$

But $$\sum_{n=0}^{m}\frac{(m-1)!mn}{n!(m-n)!(m-1)}=\frac{m}{(m-1)}\times\sum_{n=0}^{m}\frac{(m-1)!n}{n!(m-n)!}=\frac{m}{m-1}2^{(m-1)}$$

from equation 33

So $$2^{m-2}=2^m-2^{m-1}-\frac{m}{m-1}2^{m-1}+\sum_{n=0}^{m}\frac{n^2(m-2)!}{n!(m-n)!}$$

Therefore

$$\sum_{n=0}^{m}\frac{n^2(m-2)!}{n!(m-n)!}=2^{m-2}-2^m+2^{m-1}+\frac{m}{m-1}2^{m-1}=\frac{2^m}{4}-2^m+\frac{2^m}{2}+\frac{m2^m}{2(m-1)}$$

$$=\frac{2^m}{4}\left(\frac{m+1}{m-1}\right) \qquad (37)$$

Dividing equation 37 by $(m-2)!$ and multiplying by 4

$$\sum_{n=0}^{m}\frac{4n^2}{n!(m-n)!}=\frac{(m+1)}{(m-1)!}2^m=\frac{(m^2+m)2^m}{m!} \qquad (38)$$

Equation 24 then can be rewritten using equations 26, 34, and 38 in equation 24 giving

Total most probable error, per cent

$$=\frac{100(R_S+R_L+2R_N)}{2mR_A}(m^2-2m^2+m^2+m)$$

$$=\frac{100[(R_S+R_L)+2R_N]}{2R_A} \qquad (39)$$

Appendix V. Determination of Probable Error Caused by Other Customers' Loads (See Figure 5)

Excluding the effect of a single load R_A of the customer under consideration, it follows from Appendix II that the voltage error at the meter in per cent is

$$\frac{100\mathbf{I}_0(R_S+R_L+2R_N)/2}{(\mathbf{E}_{10}+\mathbf{E}_{02})/2} \qquad (40)$$

$$\mathbf{I}_0=\mathbf{E}_{02}(m-2n)/R_A \text{ where } E_{02}\approx\frac{E_{10}+E_{02}}{2} \qquad (41)$$

Then from equations 40 and 41 voltage unbalance error in per cent $=\dfrac{100R_S+R_L+2R_N}{2}\,\dfrac{m-2n}{R_A}$ approximately.

From equation 21, Appendix III, the probability of n loads on one specified side is $\dfrac{m!}{2^m n!(m-n)!}$ and the probability of n loads on either side is therefore

$$\frac{m!}{2^{m-1}n!(m-n)!} \qquad (42)$$

The probable per-cent registration error is therefore

$$\frac{100(R_S+R_L+R_N)}{2R_A}\,\frac{m!}{2^{m-1}}\sum_{n=0}^{m/2}\frac{m-2n}{n!(m-n)!} \text{ for even values of } m \qquad (43A)$$

$$\frac{100(R_S+R_L+2R_N)}{2R_A}\,\frac{m!}{2^{m-1}}\sum_{n=0}^{(m-1)/2}\frac{m-2n}{n!(m-n)!} \text{ for odd values of } m \qquad (43B)$$

Both equations 42 and 43 would have probable values of zero if summed over the range $n=0$ to $n=m$, indicating that the registration error is equally likely to be plus or minus. When summed over positive values of $(m-2n)$ the probable magnitude of the error is indicated. The upper summation limits are different for odd and even values of m since n must be an integer.

Expansion of both summations gives

$$\frac{m!}{2^{m-1}}\sum_{n=0}^{m/2 \text{ or } (m-1)/2}\frac{m-2n}{n!(m-n)!}=1\times\frac{3}{2}\times\frac{5}{4}\times\frac{7}{6}\times\frac{9}{8}\text{ to }\left\{\begin{matrix}m/2 \text{ terms for even values of } m\\(m+1)/2 \text{ terms for odd values of } m\end{matrix}\right\} \qquad (44)$$

By inserting the factors $\frac{2}{2}, \frac{4}{4}, \frac{6}{6}\ldots$ in alternate terms this expansion can be broken down.

$$=\left(1\times\frac{3}{2}\times\frac{5}{4}\times\frac{7}{6}\ldots\right)$$

$$=\left(1\times\frac{2}{2}\times\frac{3}{2}\times\frac{4}{4}\times\frac{5}{4}\times\frac{6}{6}\times\frac{7}{6}\ldots\ldots\right)\text{ to } m \text{ terms}$$

$$=\begin{cases}\dfrac{mm!}{(2^2\times4^2\times6^2\ldots m^2)} & \text{for even values of } m\\[2ex]\dfrac{m!}{[2^2\times4^2\times6^2\ldots(m-1)^2]} & \text{for odd values of } m\end{cases}$$

$$=\begin{cases}\dfrac{mm!}{2^m\left(\dfrac{m}{2}!\right)^2} & \text{for even values of } m \qquad (45A)\\[3ex]\dfrac{m!}{2^{m-1}\left(\dfrac{m-1}{2}!\right)^2} & \text{for odd values of } m \qquad (45B)\end{cases}$$

So the probable percentage error, either plus or minus, is

$$\frac{100(R_S+R_L+R_N)}{2R_A}\times\begin{cases}\dfrac{mm!}{2^m\left(\dfrac{m}{2}!\right)^2} & \text{for even values of } m \qquad (46A)\\[3ex]\dfrac{m!}{2^{m-1}\left(\dfrac{m-1}{2}!\right)^2} & \text{for odd values of } m \qquad (46B)\end{cases}$$

It will be observed that the multiplier for even values of m is the same as that for the next lower odd value of m.

A more convenient formula for determining the value of the multiplier can be developed with the use of Stirling's Formula

$$\sqrt{2k\pi}\left(\frac{k}{e}\right)^k < k! < \sqrt{2k\pi}\left(\frac{k}{e}\right)^k\left(1+\frac{1}{12k-1}\right) \tag{47}$$

Substituting the left-hand term of equation 47 in equation 45(A) and letting $m=k$

$$M=\frac{mm!}{2^m\left(\frac{m}{2}!\right)^2}=\frac{m\sqrt{2m\pi}\left(\frac{m}{e}\right)^m}{2^m\,\frac{2m\pi}{2}\left(\frac{m}{2e}\right)^m}$$

$$=\sqrt{2m/\pi}\text{ for even values of }m \tag{48A}$$

This is about 4 per cent high at $m=6$, less than 1 per cent high at $m=40$.

The use of the correction factor $[1+1/(12k-1)]$ on the right-hand side of equation 47 enables almost exact results to be obtained.

Applying this factor to numerator and denominator of 45(A) gives

$$M=\sqrt{2m/\pi}\left[\frac{1+\frac{1}{12m-1}}{\left(1+\frac{1}{6m-1}\right)^2}\right]$$

$$=\sqrt{2m/\pi}\left[\frac{\frac{12m}{12m-1}}{\left(\frac{6m}{6m-1}\right)^2}\right]$$

$$=\sqrt{2m/\pi}\left(\frac{12m}{12m-1}\right)\left(\frac{36m^2-12m+1}{36m^2}\right)$$

$$M=\sqrt{2m/\pi}\,\frac{36m^2-12m+1}{(36m^2-3m)}\approx$$

$$\sqrt{2m/\pi}\left(1-\frac{1}{4m}\right) \tag{48B}$$

Using this equation, excellent results are obtained for all even values of m. (Odd values of m give the same multiplier as the next higher even value.)

Table II applies when the customer under consideration has a single unbalanced load unit connected to the system. If this customer has a number of load units in service, the multiplier becomes smaller. This is so because the error caused by other customers' unbalances is a smaller part of his total registration. As given in equation 42, the probability of connecting q loads on either one line or the other is $p!/[2^{p-1}q!(p-q)!]$ where p is the total number of loads. This value must be multiplied by $(p-2q)/p$, the factor representing the unbalanced portion of the load. The additional factor to be applied to Table II is, therefore, the summation

$$\sum_{q=0}^{p/2}\frac{p-2q}{p}\,\frac{p!}{2^{p-1}q!(p-q)!} \tag{49}$$

for even values of p.

This can be rewritten as $\frac{1}{p}\,\frac{p!}{2^{p-1}}\sum_{q=0}^{p/2}\times\frac{p-2q}{q!(p-q)!}$ which is seen to be M/p where M is the multiplier in Table II. (Odd values of p are obviously handled in the same way as in the summations involving m and n.) Table III gives the values of factor F for various values of p.

The total most probable error caused by other customers' unbalance is, therefore, equal to the product of E, M, and F.

$$\text{Total most probable error}=\text{E}\times\text{M}\times\text{F} \tag{50}$$

where $E=\frac{100(R_S+R_L+2R_N)}{2R_A}$ the same as equation 39

M = multiplier given in Table II for any number (m) of other customers' basic load units connected to the same transformer

F = multiplier given in Table III for any number (p) of basic load units which the customer himself has connected.

Discussion

F. H. Busch (General Electric Company, Lynn, Mass.): E. C. Wentz and A. J. Petzinger's paper is a very thorough and commendable analysis of the fundamental accuracy of single-phase 3-wire metering. Combining the probability of unbalance with the error due to unbalance provides a very practical and clear method of stating the total possible error for all possible unbalances.

In commenting on the application of their analysis of the 3-wire single-phase meter to the polyphase meter with 3-wire elements, the authors state that for the 2-element meter for 3-phase 4-wire delta circuits, and for the "$2^1/_2$ element" meter for 3-phase 4-wire Y circuits, "The same method of analysis can be used, and the conclusions may safely be expected to be comparable; that is, errors probably will not exceed tenths of a per cent."

It should be pointed out that this conclusion overlooks one quite important consideration in the particular case of the $2^1/_2$ element meter for 3-phase 4-wire Y circuits. For this particular meter, the largest possibility for error arises when the line voltages are unbalanced. Such unbalance can be due to unbalanced loads on the circuit being metered, and the amount and effect probably will be small, just as analyzed by the authors. However, the unbalance also can be due to unbalanced voltage on the 3-phase distribution system supplying the transformer bank being metered. Significant errors may result even though the metered load is well balanced. For example, a 1-per-cent voltage unbalance may cause approximately 0.3% error for a balanced load on the meter.

Extra caution must be exercised to insure satisfactory voltage balance of the distribution system in the application of the $2^1/_2$ element meter. This unbalance of primary voltage supply may arise in a particular distribution system due to unequal single-phase branch loads supplying residential areas and may continue to exist for long periods of time. Such an unbalanced load, particularly for rural distribution systems, may be a definite part of the load growth plan, and, therefore, not lend itself to the probability treatment used by the authors.

E. C. Wentz and A. J. Petzinger: The only reason that the voltages in a 3-phase 4-wire system should be unbalanced is that someone somewhere on the system is overloading one phase. This could, of course, be occurring on the primary side or on the secondary side. Generally speaking, it seems that the overloading of one phase to the extent where 1-phase voltage would be appreciably reduced would be a temporary or random phenomena, and not a continued condition. In so far as this unbalance is a random phenomena, it will average out to be zero in the long run, and no customer has anything to be concerned about. If it is known in a particular installation that unbalance is a permanent condition and if the unbalance is actually large enough to affect the registration of the meters, some correction should be calculated and applied to the meter readings. It seems, however, that this will rarely be the case, and if it should be discovered to be the case, a more efficient alternate procedure would be to redistribute the loads on the lines so that they are more nearly balanced and so that the voltage is substantially balanced. The same thing can be said to be true of the 3-wire circuit; that is, if there is a permanent known unbalance in loading, the registration can be corrected for the error or the loads can be redistributed.

However, as a general conclusion, the only thing that can be said with certainty is that unbalances are random phenomena and, in the long run, are equally likely to be on any line, and the effect of one customer's unbalance on another customer averages out to be zero.

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